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A COMPARISON OF BANDWIDTH AND KERNEL FUNCTION SELECTION IN GEOGRAPHICALLY WEIGHTED REGRESSION FOR HOUSE VALUATION

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ABSTRACT

The study examines the influence of four spatial weighting functions and bandwidths on the performance of geographically weighted regression (GWR), including fixed Gaussian and bi-square adaptive kernel functions, and adaptive Gaussian and bi-square kernel functions relative to the global hedonic ordinary least squares (OLS) models. A demonstration of the techniques using data on 3.232 house sales in Cape Town suggests that the Gaussian-shaped adaptive kernel bandwidth provides a better fit, spatial patterns and predictive accuracy than the other schemes used in GWR. Thus, we conclude that the Gaussian shape with both fixed and adaptive kernel functions provides a suitable framework for house price valuation in Cape Town.

Keywords: Global model; Geographically weighted regression; House price; Kernel function

1. INTRODUCTION

Global hedonic ordinary least squares (OLS) models have, over the years, been identified and utilized for a variety of purposes in different fields. In the housing and related fields, these techniques are typically used to identify the marginal contribution of each of the housing features to price for over 50 years. Nonetheless, the global techniques are affected by their inability to completely remove spatial dependence and spatial heterogeneity in the data. These glitches, if ignored, might result in biased and unreliable parameter estimates. Des Rosier and Thériault (2008) reported that creating appropriate market segments, transforming the data, ensuring adequate model specification and applying the right spatial models are possible ways of dealing with these limitations. Of particular interest in this study is the use of spatial models to control spatial dependence and spatial heterogeneity. The models are based on refined hedonic techniques devoid of parametric restrictions with built-in features that adequately capture spatial autocorrelation (dependence) and/or variation or non-stationarity (heterogeneity) in housing prices.

Though data driven, spatial models are problem-specific solvers (tackling autocorrelation or heteroskedasticity) and do, however, have the potency to reduce other glitches found in the property market. For instance, local regression methods, such as geographically weighted regression (GWR), designed to, among other functions, control spatial heterogeneity have been found to reduce spatial autocorrelation in residual errors (McCluskey & Borst, 2011). However, despite its capability of controlling spatial heterogeneity in the property market, there is little

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attention given to GWR in the literature (Bitter et al., 2007), particularly from a pan-African perspective.

Thus far, GWR has been used in a number of housing price studies, including those published by Bitter et al. (2007), Páez et al. (2008), Borst and McCluskey (2008), Lockwood and Rossini (2011), McCluskey et al. (2013), and Bidanset and Lombard (2014). To date, the only known pan-African housing study involving GWR was undertaken by Yacim and Boshoff (forthcoming) in Cape Town, South Africa. This study is considered more comprehensive in terms of house price analysis because of the differing number of kernel functions and other schemes employed within its GWR framework.

GWR attempts to capture spatial variation in the interactions between the response variable and the different explanatory variables at each regression point in the study area, assigning weights to all observations relative to their distance from the regression point. Accordingly, the nearer an observation is to the regression point, the more the weights assigned thereby exert more of an effect on the regression estimates than more distant observations. Kernel (density) function and bandwidth schemes are central to the effective performance of GWR. Thus, the optimal performance of GWR in house price estimation is a reflection of the proper selection of kernel density and bandwidth, and their parameter settings. According to Bitter et al. (2007) and Guo et al. (2008), a higher bandwidth will produce coefficient estimates that are similar to estimates of the global OLS models with a spatial pattern that appears smooth across the geographic space of the local market. Contrariwise, if a lower bandwidth is used, the coefficient estimates will only be for observations that are closer to the regression points, thereby causing a high variance (Fotheringham et al., 2000; Fotheringham et al., 2002). To ameliorate the glitches, and because house data behave differently relative to geographical location, different GWR schemes were tested with South African data to unravel the best kernel and bandwidth specifications.

Previously, the study of Bidanset and Lombard (2014) examined the combined contribution of bandwidths and kernel functions to a house price analysis conducted in Norfolk, Virginia. However, the main goal of this study is to see if a data example derived from South Africa (a region with different socioeconomic and contextual settings that influence buyers' attitudes) might replicate the results of previous housing price analysis. The findings of this study could be of great interest to analysts and modelers as it provides an easy framework for selecting optimal kernel function and bandwidth without the need to try different schemes in the GWR assessment.

2. METHODS

The GWR (one variant of the OLS techniques) used the location coordinates (u, v, also depicted as i) of the observations to vary the parameter estimates locally across the geographic area. This is represented as

$$Y = \beta(u,v) + \sum_{s=1}^{s} X_s \beta_s (u,v) + \varepsilon \tag{1}$$

where Y is the vector of the response variable at location u, v, which is regressed against a set of explanatory variables X, the parameters for the regression coefficient, given as β and the random error term ε. The GWR utilizes a weighted least squares (WLS) technique to weight the observation of a house at point i (subject house) relative to the distance from its nearest neighbors. Thus, the WLS technique allows more local than global parameter estimates to be calculated across locations in geographical space, thereby producing an output that represents non-stationarity. Simply stated, the GWR is a combination of a number of small weighted hedonic OLS that are performed around each subject house (Moore & Myers, 2010). According to Griffith (2008), the normal probability model is the known approach used to draw inferences from the WLS technique. The specification of weight with the WLS method is given as

$$\hat{\beta}(u,v) = \left[X^T W(u,v) X\right]^{-1} X^T W(u,v) Y \tag{2}$$

The weight matrix $W(u, v)$ is taken as a diagonal matrix where each element $W_{jj}(u, v)$ defines the geographic area for every location and $X^T W(u, v) X$ denotes the geographically weighted variance-covariance matrix. The matrix $W(u, v)$ housed the geographical weights in its main diagonal and 0 element in its off-diagonal location

$$\begin{bmatrix} w_1(u,v) & 0 & 0 & 0 \\ 0 & w_2(u,v) & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & w_n(u,v) \end{bmatrix} \tag{3}$$

Leung et al. (2000) reported that in using the WLS technique, the generated estimators at point i were obtained by solving the optimization problem. The solution was simply to find the β_0, β_1, β_2, β_3, …, β_s at each location such that

$$\sum_{j=1}^{n} w_j(u,v)(y_j - \beta_0 - \beta_1 x_{j1} - \cdots - \beta_s x_{js})^2 \tag{4}$$

was minimized. Given the right choice of weights $w_j(u, v)$, which is a function of the locations at which observations are made, different emphases can be given to different observations to produce the estimated parameters at point i.

2.1. Specification of Weights in GWR

In GWR, the weights are assigned relative to their distance from the regression point. Generally, spatial analysis observations that are closer to the regression points are expected to have a greater influence on the parameter estimates than distant observations, thus obeying the distance decay principle (Figure 1) and Tobler's (1970) first law of geography in which *"…near things are more related than distant things."* The spatial kernel (density) function is the scheme in GWR that isolates the neighborhood of observations, thus facilitating the creation of a regression at every point of the coordinate system and producing a vector of parameters specific to each location (Cho et al., 2010). The spatial kernel function $[K(d_{ij}/d_{max})]$ is used to map the area of observations through $W(u, v)$, generating n segments of observations. Consequently, a vector of parameters is assessed for every observation, producing $n\,\hat{\beta}(u, v)$'s, and for all $d_{ij} \geq d$, $K(d_{ij}/d_{max}) = 0$. According to Cho et al. (2010), a number of possibilities exist for K; nonetheless, it must be real, continuous, bounded, symmetric and integrated to 1 in the same way the kernel function is used in non-parametric regression (Cameron & Trivedi, 2005).

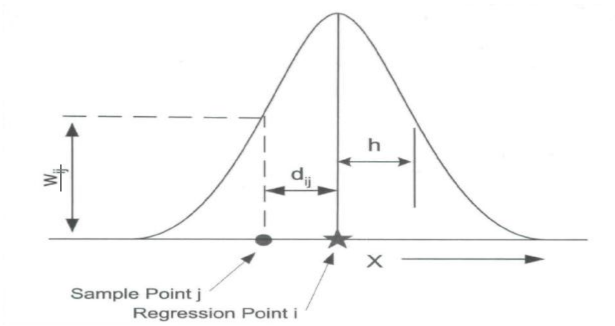

Figure 1 A two-dimensional spatial weighting kernel (adopted from Borst & McCluskey, 2008, p. 35)

There are two accepted spatial weighting kernel functions, including fixed and adaptive kernels, used to provide the weights in the model. The fixed kernel function is used to define the geographical weight matrix W with the assumption that the bandwidth at each house location i is a constant across the geographic space. The adaptive kernel function assumes that the bandwidth adapts itself in size relative to variations in the density of the data. In an adaptive kernel specification, a larger bandwidth is applied only when the data is sparse; conversely, a smaller bandwidth is applied only when the data is dense. Thus, to achieve a reasonable degree of fit, the analyst or modeler must be more careful in the selection of a bandwidth than the shape of the spatial kernel function. There are two acceptable shapes the kernel function must take: Gaussian and bi-square. Depending on the shape the analyst or modeler chooses, this shape might either take the form of fixed or adaptive kernel functions. Therefore, a fixed Gaussian and a fixed bi-square and an adaptive Gaussian and an adaptive bi-square are possibilities that can be applied in GWR. Accordingly, the fixed Gaussian-shaped function is given as

$$w_{ij} = \exp\left\{-\left(\frac{d_{ij}}{h}\right)^2\right\} \tag{5}$$

the fixed bi-square-shaped function is given as

$$\begin{aligned} w_{ij} &= \left[1-\left(\frac{d_{ij}}{h}\right)^2\right]^2 \quad && when\, d_{ij} < h, \\ w_{ij} &= 0 && when\, d_{ij} > h \end{aligned} \tag{6}$$

the adaptive Gaussian-shaped function is given as

$$w_{ij} = \exp\left\{-\left(\frac{d_{ij}}{h_{i(k)}}\right)^2\right\} \tag{7}$$

and the adaptive bi-square-shaped function is given as

$$\begin{aligned} w_{ij} &= \left[1-\left(\frac{d_{ij}}{h_{i(k)}}\right)^2\right]^2 \quad && when\, d_{ij} < h_{i(k)}, \\ w_{ij} &= 0 && when\, d_{ij} > h_{i(k)} \end{aligned} \tag{8}$$

The study of Borst and McCluskey (2008) used a two-dimensional representation of the kernel function to explain the concept of the fixed and adaptive weightings in Figure 1. The height of the curve at point j, provided by W_{ij}, is the weight applied to point j when point i is the regression point and d_{ij} is the distance between the regression point i and data point j. The spatial kernel bandwidth denoted by a scalar quantity, which can be fixed (h) or adaptive ($h_{i(k)}$), is the parameter that affects how weight is calculated relative to the change in distance between the regression point and observation (sample) points. The k^{th} term in the adaptive bandwidth size is used to define the nearest neighbor distance. Three procedures are used to supply the bandwidth, with the first two being (*i*) the supply from the analyst or modeler and (*ii*) the estimation derived using the cross-validation (*CV*) technique. This technique estimated a bandwidth that minimizes the *CV* score of:

$$CV = \sum_{i=1}^{n} \left[Y - \hat{Y} \neq i(h) \right]^2 \tag{9}$$

where $\hat{Y} \neq i\ (h)$ is a predictor at point i using example data that excludes point i from the estimation. The likelihood of over-fitting the model and *CV* being an indicator of goodness of fit are the main drawbacks of the *CV*, while procedure (*iii*) determines a bandwidth that minimizes the Akaike Information Criteria (*AIC*), providing a balance between goodness of fit and degree of freedom. The *AIC* utilized via the golden search scheme are used in this analysis. The golden search selection criterion makes it possible for the optimal bandwidth size to be determined automatically, thereby avoiding the problem of over-fitting the local model. The *AIC* are given as:

$$AIC = 2n \log(\hat{\sigma}) + n \log(2\pi) + n \left\{ \frac{n + tr(Z)}{n - 2 - tr(Z)} \right\} \tag{10}$$

where n is the total number of observations, the expression $tr(Z)$ is the trace of the hat matrix and $\hat{\sigma}$ denotes the estimated standard deviation of the residuals as:

$$\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \left(Y - \hat{Y} \right) / \left(n - 2tr(Z) + tr(Z^T Z) \right)} \tag{11}$$

and Z is the hat matrix that defines each row as

$$Z_i = X_i \left[X^T W(u,v) X \right]^{-1} X^T W(u,v) \tag{12}$$

Five models are estimated in this study, including (i) the global OLS as the baseline (model 1) and the GWR with different specifications, namely (ii) fixed kernel bandwidth–Gaussian shape (model 2), (iii) fixed kernel bandwidth–bi-square shape (model 3), (iv) adaptive kernel bandwidth–Gaussian shape (model 4), and (v) adaptive kernel bandwidth bi-square shape (model 5).

2.2. Data

The example data is comprised of the sales transactions of single-family houses sold between January 2012 and May 2014 in Cape Town, South Africa. To ameliorate any potential errors that emerge as the consequence of distorted market forces, only the arm’s length sales listed in the market were considered. The city valuation office (CVO) of Cape Town was the primary source of the information used in this analysis. The CVO prepares the general valuation roll for each municipality within its jurisdiction every three years and administers taxes based on the assessed value of all properties. The data supplied contains a total of 46 information attributes on each of the 3.526 property observations. Outliers, incomplete transactions and transactions other than single-family houses were removed from the analysis, reducing the number to 3.232 observations. In addition, the number of attributes was pared down to avoid multicollinearity and dimensionality problems that might reduce the strength of the models. The process was considered adequate because when a preliminary assessment was performed via the regression techniques, the selected attributes were found to be significant at $p < 0.05$. The procedure, coupled with principal component analysis, was used by Bitter et al. (2007) in the selection of variables. Similarly, Páez et al. (2008) partly used the regression techniques, correlation analysis and multicollinearity tests to select the variables used in their analysis.

Though no existing economic idea underpinned a particular functional form of the hedonic regression models, a prior study with the Cape Town housing data conducted by Yacim and Boshoff (2018) suggested the semi-log form performs better in pricing houses than the linear and log-log forms. Consequently, the response variable (assessed value) is transformed into its natural logarithm form so that the interpretation of coefficients could be provided in percentages. The explanatory variables included structural, temporal (X) and location or geo-coordinates (x, y or u, v (see Equation 1)), reflecting the spatial characteristics of houses as follows:

- Size: house size in square meters (continuous)
- Coordinate values: easting (x-coord) and northing (y-coord)
- Beds: number of bedrooms (categorical)
- Quality: house quality (categorical)
- Condition: house condition (categorical)
- Month: reverse month of sale (continuous)
- View: house view (categorical)
- Style: house building style (categorical)
- Floors: number of stories in the house (categorical)
- Pool: swimming pool in square meters (continuous)

The presence of geo-coordinates (x, y) within the Cape Town house data made it possible for the local GWR model to be used in this analysis. Utilizing this model within the global OLS models requires some form of interaction between the coordinates and/or house attributes (see, for example, Thériault et al. (2003); Bitter et al. (2007)). Table 1 presents the descriptive statistics of the variables used in this analysis. The mean assessed value is about R4.5m, while the lowest assessed value for a house in the sample is R824.000 and the maximum assessed

Table 1 Descriptive statistics of variables

Variable	Mean	Std. dev.	Min	Max
Log of assessed value	15.1543	0.53958	13.62	17.45
Assessed value	4483474	3117754	824000	38000000
Beds	3.56	0.992	1	10
Quality	3.49	0.615	1	6
Condition	3.51	0.627	1	5
Month	14.88	8.165	1	29
View	3.58	0.963	1	6
Style	3.03	0.430	1	7
Floor	1.52	0.553	1	3
Size	177.48	78.905	31	599
Pool	13.97	18.382	0	154

value for a house is R38.000.000. The smallest house in the sample is 31 m^2 and the largest house is 599 m^2. Again, a typical house in the sample comprises an average of four bedrooms.

3. RESULTS AND DISCUSSION

The results and discussion of all models are contained in this section. As previously noted, the baseline is the global OLS models that used a single equation for the entire jurisdiction. The baseline model used the structural and temporal variables given in Table 1 and in addition, reflects the location element with geo-coordinates in the form of a second-order polynomial expansion. The GWR used the geo-coordinates directly to measure the distance between the houses in the study area. Table 2 provides a summary of the goodness of fit and a performance evaluation of all models.

Utilizing the x, y coordinates has greatly improved the goodness of fit from the adjusted R^2 of 37% to 58% in the global model. Consequently, all variables are significant with the appropriate signs (see Table 3). However, the four non-stationarity models with differing kernel functions and bandwidths reveal that the Gaussian shape with both fixed and adaptive kernel functions performed in an optimal manner. Specifically, the best predictive power is consistently achieved by model 4, the adaptive kernel bandwidth with a Gaussian shape, in terms of the mean absolute error (*MAE*), the root mean squared error (*RMSE*), the coefficient of dispersion (*COD*), price-related differentials (*PRD*) and *AIC*.

Table 2 Goodness of fit and performance evaluation measures

Measure	Model 1	Model 2	Model 3	Model 4	Model 5
R^2	0.5820	0.7598	0.6489	0.7802	0.7487
Adj. R^2	0.5802	0.7289	0.6356	0.7432	0.7228
AIC	2394	1171	1995	1005	1202
MAE	1248596	961721	1154959	907498	970154
RMSE	2244305	1829536	2109950	1719293	1759364
COD	0.266	0.192	0.237	0.183	0.201
PRD	1.140	1.085	1.120	1.082	1.088

The results produced by the Gaussian-shaped kernel functions are consistent with those of Bidanset and Lombard (2014) for Norfolk, Virginia. However, while the adaptive kernel performed optimally in this study, the fixed kernel was optimal in their study. Relative to the global model, all GWR models provide a better fit for the data, revealing the influence of spatial patterns and separate regression equations versus calibrating a single equation for the entire sample area. The better fits (higher R^2 (Berawi et al., 2010)) generated imply the ability of GWR to effectively explain variations in Cape Town house prices. The correlation between data points is highly significant, resulting in higher and more accurate estimations of house prices in GWR models. Thus, the results suggest that it is better to calibrate a separate equation for each of the houses within a geographic area (which the GWR is designed to accommodate) than to calibrate one equation (OLS) for the entire location. The regression results for model 4 (the best in this analysis) are shown in Table 4. The results of the other schemes utilized in this study are presented in the appendix since the concern here is to report on the best-performing model. The difference of 16.3% in the adjusted R^2 between the global model 1 and model 4 in Table 2 is quite high, revealing the need to use GWR for house valuation in the city of Cape Town.

Table 3 Regression coefficient for global model 1

Variable	Coefficient	Std. Error	t-statistic	Probability
CONSTANT	-12300.6	536.116	-22.9439	0.00000
BEDS	0.059718	0.007338	8.13798	0.00000
QUALITY	0.132334	0.014555	9.09195	0.00000
CONDITION	0.067822	0.014018	4.8383	0.00000
FLOOR	0.141224	0.012929	10.9231	0.00000
STYLE	-0.08880	0.014496	-6.12601	0.00000
VIEW	0.093859	0.006858	13.6852	0.00000
MONTH	0.001473	0.000755	1.95073	0.05118
SIZE	0.002467	9.8917e-01	24.9435	0.00000
POOL	0.002169	0.0003742	5.79546	0.00000
X	-0.001557	0.0003261	-4.77612	0.00000
Y	-0.006530	0.0002843	-22.9722	0.00000
X^2	5.43697e-01	5.92997e-01	9.16864	0.00000
Y^2	-8.65728e-01	3.76902e-01	-22.9696	0.00000
XY	-4.20318e-01	8.57735e-01	-4.90033	0.00000

Table 4 reveals the varying parameter estimates of model 4 – the GWR at each of the 3.232 observation points, defined by their lower (lwr) quartile, upper (upr) quartile, minimum, maximum and median values. In particular, the importance of the minimum and maximum values reflects variations in housing behavior from one segment of Cape Town to the other and are thus counterintuitive (Bitter et al., 2007) in some instances. For instance, a story house has estimates that range from a minimum value of -0.26% at one location to a maximum value of 0.37% at another location.

Table 4 GWR coefficient for with adaptive kernel bandwidth – Gaussian shape (model 4)

Variable	Minimum	Lwr Quartile	Median	Upr Quartile	Maximum
Intercept	11.440575	13.148830	13.512586	14.032791	16.156946
BEDS	-0.161825	0.023395	0.045977	0.070476	0.181461
QUALITY	-0.122592	0.082467	0.130417	0.172325	0.340027
CONDITION	-0.332795	-0.015389	0.051746	0.102259	0.357772
FLOOR	-0.261197	0.136058	0.198052	0.245162	0.373696
STYLE	-0.519821	-0.126048	-0.037687	0.005851	0.279496
VIEW	-0.018342	0.035973	0.055040	0.081278	0.344379
MONTH	-0.006675	-0.001328	0.001475	0.003590	0.006818
SIZE	0.000532	0.002069	0.002457	0.003042	0.005399
POOL	-0.003314	0.000391	0.002031	0.003517	0.028744

The implication of this variation is that *ceteris paribus*, a house with two or more floors sells for 26% less at one location than a similar house type that sells for 37% more at another location. The negative values in the coefficients of house quality and house condition are also counterintuitive and might reflect buyers' attitudes toward home purchases in which houses of poor condition or quality are purchased for more at one location than those of good or excellent condition or quality in other locations of the study area. Interestingly, the parameter estimates for house size reveal a positive and smooth spatial pattern in the Cape Town housing data.

4. CONCLUSION

The global OLS models produce regression coefficients that do not reflect the true relationship within housing datasets because of their limitations in correcting for spatial effects. GWR permits local variation of parameter estimates within a geographic region, thereby producing reliable results. However, the optimal performance of GWR is predicated on the choice of spatial weights kernels and bandwidths. Providing a framework for analysts and modelers, particularly within a pan-African context, is the main motivation, among others, of this study. Accordingly, the study compares the performance of different spatial kernel bandwidth weighting specifications in GWR relative to the global models using an example of housing data from Cape Town, South Africa. Specifically, the *AIC* for the golden bandwidth search scheme on fixed (Gaussian and bi-square) and adaptive (Gaussian and bi-square) kernel functions were used.

While all GWR models improve upon the results of the stationary coefficient global models, despite the inclusion of the second-order polynomial location coordinates, the adaptive kernel bandwidth–Gaussian shaped GWR (model 4) outperformed all other specifications in this study. The fixed kernel bandwidth–Gaussian shaped GWR (model 2) trailed closely behind, revealing that Gaussian-shaped GWR is suitable for house price valuation in Cape Town. One notable relationship that the results of this study shares with those of Bidanset and Lombard (2014) is the fact that the Gaussian-shaped scheme with fixed and adaptive kernels is optimal. However, while this study found the adaptive kernel to perform best, their study found the fixed kernel to be optimum. Thus, analysts and modelers should consider the use of the Gaussian-shaped scheme with either fixed or adaptive kernel functions in their assessments of house prices, as suggested in both studies. Additionally, the results provide complete evidence that either of the spatial weights specifications in GWR is a viable alternative in situations where price estimation is the principal interest.

One area of concern is the high COD and PRD produced by the models used in this analysis. This result might be the consequence of inaccurate data collection, specification errors exacerbated by omitted attribute information or market inefficiencies. Further research might be necessary to unravel the causes before a definite position can be reached regarding the Cape Town housing data.

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Appendix

Table A1 GWR coefficient for with fixed kernel bandwidth – Gaussian shape (model 2)

Variable	Minimum	Lwr Quartile	Median	Upr Quartile	Maximum
Intercept	11.749901	13.141726	13.472179	13.877878	16.365673
BEDS	-0.503970	0.031982	0.051594	0.077907	0.170642
QUALITY	-0.346144	0.111413	0.146989	0.170931	0.342235
CONDITION	-0.326445	-0.003303	0.053753	0.089996	0.500891
FLOOR	-0.306870	0.128274	0.193511	0.241548	0.661929
STYLE	-0.551050	-0.129115	-0.048300	-0.009109	0.163901
VIEW	-0.056272	0.048667	0.074195	0.097546	0.512911
MONTH	-0.041947	-0.000844	0.001395	0.003389	0.008266
SIZE	-0.003287	0.002226	0.002536	0.002875	0.005843
POOL	-0.003402	0.000247	0.001960	0.003706	0.082986

Table A2 GWR coefficient for with fixed kernel bandwidth – bi-square shape (model 3)

Variable	Minimum	Lwr Quartile	Median	Upr Quartile	Maximum
Intercept	12.343200	13.322556	13.531922	13.749112	15.113559
BEDS	-0.068802	0.037119	0.059675	0.064017	0.116694
QUALITY	0.021001	0.130432	0.166873	0.173601	0.270686
CONDITION	-0.247222	0.010539	0.024368	0.051748	0.239120
FLOOR	0.041668	0.163349	0.185753	0.205303	0.394121
STYLE	-0.360157	-0.120779	-0.071933	-0.039892	0.036840
VIEW	0.009687	0.049372	0.102480	0.151051	0.173738
MONTH	-0.005323	-0.000145	0.002063	0.003087	0.004750
SIZE	0.000863	0.002207	0.002489	0.002736	0.004833
POOL	-0.000199	0.001417	0.002068	0.003160	0.007763

Table A3 GWR coefficient for with adaptive kernel bandwidth – bi-square shape (model 5)

Variable	Minimum	Lwr Quartile	Median	Upr Quartile	Maximum
Intercept	12.075213	13.219495	13.567783	14.024588	15.087940
BEDS	-0.040660	0.030778	0.050246	0.074774	0.154730
QUALITY	-0.035005	0.093360	0.128665	0.170019	0.340894
CONDITION	-0.202135	-0.013169	0.046224	0.096794	0.253879
FLOOR	-0.089551	0.114697	0.202241	0.271022	0.387340
STYLE	-0.365715	-0.110069	-0.048889	-0.011873	0.109121
VIEW	-0.030503	0.038733	0.057871	0.084488	0.248299
MONTH	-0.005608	-0.000977	0.001555	0.003778	0.008745
SIZE	0.000462	0.002215	0.002485	0.003004	0.004404
POOL	-0.003693	0.001141	0.002026	0.003410	0.006709