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*Research Article*

# Fair Allocation Mechanisms for Logistic Resources in Multi-Issue Bankruptcy with Cross-Claims

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**Abstract:** This study proposes a fair allocation framework for logistics settings in which multiple agents share scarce resources, routes, or operational capacities under interdependent constraints. To this end, it applies the Multi-Issue Bankruptcy Problem with Cross-Claims (MBC) model to the field of cooperative logistics. The study shows that the MBC framework provides a rigorous way to represent allocation problems arising when multiple agents simultaneously claim the same logistics capacity across multiple states. The Constrained Equal Awards (CEA) rule is adapted and implemented within this setting through an iterative linear programming procedure. The approach is illustrated through a real case study applaied in the departament of Magdalena, Colombia, and complemented with comparative analyses under Proportional-MBC, a simplified Constrained Equal Losses (CEL)-inspired benchmark, and alternative claimant-state structures. The comparative analysis shows that the proposed CEA–MBC method achieves the highest average satisfaction rate while fully utilizing all four available states. It was also found that the Proportional–MBC method allocates a slightly larger amount of resources and attains the highest Jain's Fairness Index; nevertheless, both MBC-based approaches substantially outperform the CEL benchmark, which leaves the highest unmet demand and fully utilizes only two states. These findings demonstrate that the proposed the proposed framework yields feasible and distributively meaningful allocations while offering a useful methodological basis for designing more equitable and collaborative logistics systems.

**Keywords:** Allocation rules; Bankruptcy theory; Constrained equal awards rule; Constrained equal loses; Game theory; Logistics cooperation; Proportional MBC rule

## 1. Introduction

Resource allocation is an interesting problem in different logistics management areas. One particular problem arises when the available resources are insufficient to meet the stakeholders' demands. Such situations can be modeled in a cooperative game theory context, mainly in the area of bankruptcy problems. A good or resource is divided among several agents, as each agent has rights over the others. This situation leads to an effective and equitable allocation of resources. When contextualizing this problem, the asset allocation must satisfy two fundamental conditions. First, the agents involved should not be allocated more than they demand and, at a minimum, they are not entitled to anything; this is defined as a claim not being less than zero to ensure that there is no under-allocation. Second, the distribution must be fully demanded so that all resources are allocated. These problems have been studied by Aumann and Maschler (1985) and O'Neill (1982). Solutions to these problems can be found through different distribution rules.

Resource allocation problems in logistics are considered here, since the allocation of shared resources affects many logistics and production processes, giving rise to conflicts between the actors involved. Given the insufficiency of available resources to fully satisfy the demands of all agents, an equitable distribution mechanism that offers fairness and restoration efficiency

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for allocation is required. For this reason, bankruptcy problems with multiple issues and cross-claims arise due to situations in which agents have rights or claims to different types of limited resources, and their claims overlap in several respects. Such is the case with collaborative transport networks, storage-backed capacity allocation, and critical production and distribution planning. For example, in a collaborative logistics system, several companies may claim rights to move part of the truck fleet, use certain warehouse capacity, or use a certain amount of raw materials for production.

When the supply of resources is insufficient, determining a distribution method that is equitable and does not favor any of the agents is necessary so that they are not treated arbitrarily. One of the main challenges in these problems lies in the existence of cross-claims, that is, situations in which agents simultaneously request more than one resource and their demands affect the system's overall availability. In these cases, allocation based solely on a fixed proportion of demand can lead to inequalities, inadvertently favoring certain agents over others. Applying methods inspired by game theory and bankruptcy problems is necessary to address this issue, as these allow for a fair distribution of available resources, considering both individual claims and the interdependence between different categories of resources. In this particular case, the Constrained Equal Awards Rule (CEA) is considered an optimal solution as it provides an allocation that respects availability limits while maintaining fairness in distribution.

This article is based on the multi-issue bankruptcy with crossed claims (MBC) model presented in Acosta-Vega et al. (2022), extending it to a new field of logistics. Although the original formulation established the theoretical foundations of MBC and its potential for addressing the problem of reducing emissions of different pollutants, it had not been explored in the field of logistics. In this work, resource allocation is not simply a matter of distributing a single claim among independent agents. The logistics system presents multiple divisible resources, and each agent has simultaneous rights to several of them. This structure generates simultaneous claims because allocation in one state affects availability in others. Under these conditions, classical bankruptcy models do not solve this problem because they do not formally capture the structure of cross-claims. The MBC framework allows for the explicit modeling of this structural interaction between resources and agents, offering a rigorous representation of distributive conflict in collaborative logistics environments. Therefore, cooperative game theory offers not only a normative tool for equity but also an instrument of distributive justice in rural logistics environments characterized by structural scarcity, where the absence of formal allocation criteria can generate conflicts among actors.

In rural logistics systems, equity is not only a normative principle but also an operational condition. An equitable allocation mechanism contributes to cooperative stability among producers by reducing incentives for strategic withdrawal or conflict. This mitigates disputes over shared infrastructure, supports predictable service levels across municipalities, and improves confidence in collective distribution plans. In fragmented rural supply chains, where producers depend on shared transport and processing capacity, equitable allocation rules strengthen long-term coordination and improve the system's overall resilience under conditions of scarcity.

This study demonstrates, for the first time, how this approach can be adapted to contexts in which scarce logistics resources must be allocated among multiple agents with cross-claims. This adaptation is not a simple application but rather an extension that reveals new findings about trade-offs between fairness and efficiency in the allocation of logistics resources. Thus, the work contributes to the literature on bankruptcy problems with competing claims and to the field of logistics, positioning MBC as a practical mechanism for supporting decision-making in complex environments with limited resources.

## 2. Literature Review

The management of scarce resources is very important in areas such as administration, IT, logistics, and production, where the objective is to allocate a set of resources to a series of agents who use them to produce goods. This leads to a situation in which each agent demands the production-related resources; that is, there is a finite number of perfectly divisible resources and some agents who have claims on them whose demand exceeds the available resources. These types of bankruptcy problems have been analyzed axiomatically by Thomson (2003, 2015, 2019), who established a solid theoretical framework for distributing insufficient resources among creditors with claims exceeding what is available. Calleja et al. (2005) studied an extension of these problems, where a divisible resource must be distributed across several categories, and each agent has rights to each of them, generating a total demand that exceeds the available resource, and also introduced two random arrival rules for multi-issue allocation problems. Izquierdo and Timoner (2016) developed an alternative method of addressing multi-issue allocation problems based on the assumption that the available resource has already been divided among the different agents according to external criteria; however, claimants retain their rights to each claim.

There are many studies on the application of distribution problems, for example Pulido et al. (2002) on university management, or the work of Gallastegui et al. (2002) on the exploitation of fishery resources. Other recent studies have focused on more complex environments with multiple states, such as those addressed by Bergantiños et al. (2018, 2020) and Gutiérrez et al. (2018). Applications of distribution problems have also been made to environmental issues, where there is a wide variety of literature reviews and articles. One of the most recent and important is the study of carbon dioxide emissions distribution, which was previously studied in depth by Duro et al. (2020). Singh and Altman (2011) applied resource allocation models to bankruptcy problems in order to distribute limited transmission power equitably according to individual demands. Similarly, studies such as those by Lorenzo-Freire et al. (2010) and Bergantiños et al. (2011) extended the rules of equal allocation and equal losses, while other studies such as those by Bergantiños et al. (2009) and Moreno-Ternero (2009) proposed extensions of the proportional rule. Algaba et al. (2019) integrate a cooperative approach to the analysis of public services. Similarly, Sánchez-Soriano et al. (2016) provide a perspective on how the proportional rule can be applied to electoral systems, broadening the scope of these tools.

In cooperative game theory, the principles of distributive justice prevail across different sectors in various applications. Some examples are those of Frisk et al. (2006), Niyato and Hossain (2006), Gozálvez et al. (2012), and Lucas-Estañ et al. (2012). Similarly, in environmental management, various studies have been conducted, such as those by Giménez-Gómez et al. (2016), Gutiérrez et al. (2018), Duro et al. (2020), Wickramage et al. (2020), and Algaba et al. (2023), who studied principles of distributive equity applied to the sustainable management of natural resources. In the field of cooperative game theory, Lemaire (1984) proposed alternatives based on game theory as opposed to traditional accounting methods, while Young (1994) established a solid theoretical basis for equitably distributing costs in organizations without reference prices, giving rise to multiple practical applications. In sectors such as health, González Rodríguez (2002) investigated the allocation of costs in surgeries and referrals to the private sector. In the insurance field, equity in political representation is analyzed by Karagök (2006), and strategic analysis in transport networks is expanded upon by García-Jurado (2005). In the field of logistics, Audy et al. (2010) developed methods for sharing savings in collaborative transport agreements, while Liu and Liao (2021) optimized urban waste collection using efficiency and sustainability criteria. Maflahah et al. (2024) applied the Shapley value of cooperative game theory to form horizontal and vertical collaborative alliances, which allow the reduction of oligopoly within the salt supply chain in Indonesia, providing an empirical example of coalition formation and payoff division relevant to collaborative logistics settings.

Dinar and Hogarth (2015) highlight the contributions and challenges presented by game theory in cooperative decision-making contexts in water management; the cooperative distributive perspective has been very useful within supply chains in this approach. Similarly, Zheng et al. (2019) explicitly include criteria of distributive justice in closed-loop supply chains. Taken together, these studies show that game theory offers a solid framework for addressing equity, sustainability, and efficiency issues within collaborative economic scenarios.

Finally, there are also other variations of bankruptcy problems known as multi-issue bankruptcy problems with cross-claims, as outlined in Acosta-Vega et al. (2022), where these models are introduced, inspired by a real problem of minimizing emissions of different pollutants. Acosta-Vega et al. (2023, 2025) studied other related works on distribution rules, including one related to the restricted proportional allocation rule and the most recent one presenting a priority-based rule for this type of problem. The rest of the document is organized as follows: Section 3 presents multi-issue bankruptcy problems with cross-claims (MBC). Section 4 defines the equal distribution rule for multiissue and cross-claims bankruptcy problems. In Section 5, we present several interesting properties in the MBC problem context. Section 6 includes an application of the equal distribution rule in a logistical problem, Section 7 is the discussion of results, and Section 8 concludes.

## 3. Multi-Issue Bankruptcy Problems with Logistics Crossed Claims

A bankruptcy problem occurs when an available resource is insufficient to fully satisfy a group of agents' demands. In other words, there is a situation of scarcity in which the sum of the claims exceeds the total amount available, making it necessary to establish fair allocation rules. In logistical terms, this situation arises when transport, storage, or distribution capacity is less than the actors' aggregate demand in the chain. This type of problem has been widely studied in cooperative game theory due to the need to design consistent, equitable, and transparent distributive mechanisms.

A bankruptcy problem (BP) is a triplet $(N, E, c)$, where $N = \{1, \ldots, n\}$ is the set of claimants, with $|N| \geq 2$. $E \in \mathbb{R}^+$ is a perfectly divisible amount and $c = (c_1, \ldots, c_N) \in \mathbb{R}^N_+$ is the vector of demands, such that $C = \sum_{j \in N} c_j > E$. The set of all bankruptcy problems with set of agents $N$ is denoted by $BP^N$. The solution to a bankruptcy problem is an allocation rule that allows the amount to be reasonably distributed through a function of the agents' demands. Let $\digamma$ be a function that, for each bankruptcy problem $(N, E, c)$, associates a value $\digamma(N, E, c) \in \mathbb{R}^N_+$ satisfying equation (1) and (2):

$$\sum_{i \in N} \digamma_i(N, E, c) = E, \tag{1}$$

$$0 \leq \digamma_i(N, E, c) \leq c_i, \quad \forall i. \tag{2}$$

A multi-issue bankruptcy problem with cross-claims (MBC) can be considered as a 5-tuple, defined as $MBC = (\mathcal{M}, N, E, c, \psi)$, where $\mathcal{M} = \{1, 2, \ldots, m\}$ is a finite set of logistical resources or issues, and each issue $i$ has a perfectly divisible resource available $e_j$ (such as allocation of transport costs, warehouse inventory costs, budgets, etc.). There are $n$ logistical stakeholders or claimants $N = \{1, 2, \ldots, n\}$, where each one claims an amount $c_i$ of the resource. As shown in Figure 1, these claims involve simultaneous cross-claims. A function $\psi(i) \subseteq \mathcal{M}$ is defined, indicating the states to which claimant $i$ is associated. The objective is to allocate resources fairly $E = (e_1, e_2, \ldots, e_m)$, considering the cross-claims $c = (c_1, c_2, \ldots, c_n)$. Given a problem $(\mathcal{M}, N, E, c, \psi) \in MBC$, a feasible assignment is a vector $F(\mathcal{M}, N, E, c, \psi) \in \mathbb{R}^N_+$ satisfying equation (3) and (4):

$$0 \leq F_i(\mathcal{M}, N, E, c, \psi) \leq c_i \quad \text{for all } i \in N, \tag{3}$$

$$\sum_{i \in N : j \in \psi(i)} F_i(\mathcal{M}, N, E, c, \psi) \leq e_j \quad \text{for all } j \in \mathcal{M}. \tag{4}$$

We denote by $A(\mathcal{M}, N, E, c, \psi)$ the set of all feasible allocations by denote. Condition 1 indicates that each agent receives at most their entitlement, but less than nothing. Condition 2 implies that no claimant can receive more than they demand.

An MBC rule is an assignment $F$ that associates with each $(\mathcal{M}, N, E, c, \psi) MBC$ unique and feasible assignment $F(\mathcal{M}, N, E, c, \psi) \in A(\mathcal{M}, N, E, c, \psi)$.

## 4. Constrained Equal Award Rule for MBC Problems

In bankruptcy problems, the Constrained Equal Awards (CEA) rule is one of the classic mechanisms of distributive justice. Its principle states that all claimants should receive the same allocation whenever possible while simultaneously respecting two conditions: (1) no claimant can receive more than they claim, and (2) the total allocation cannot exceed the available resources. In this sense, the CEA rule satisfies the principle of equal treatment, meaning that all claimants receive the fairest possible treatment under conditions of scarcity.

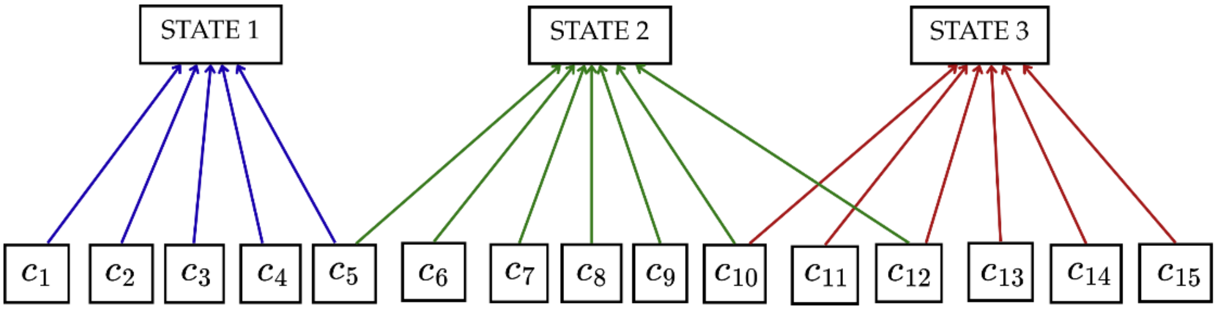


**Figure 1** An example of a multi-issue bankruptcy problem with cross-claims (MBC)

This equality principle must be adapted to a more complex structure in the context of multi-issue problems with cross-claims, where availability depends on multiple interrelated resources. The CEA rule is extended to the MBC framework through an iterative linear programming procedure. At each stage, a common maximum feasible allocation is determined for the set of active claimants, simultaneously considering capacity constraints in all involved states. When claims involve multiple states simultaneously, all active claimants receive the same feasible allocation at each stage, preserving equality as long as the constraints allow. When a claimant's demand is fully satisfied or a state's capacity is exhausted, the active set is updated, and the procedure continues until no further allocation is feasible.

In single-cause insolvency proceedings, "maximum possible equitable distribution" means that no creditor can receive more than those with lower claims, unless the latter have received their entire claim. This can be mathematically expressed as follows (Equation 5):

$$CEA_i(N, E, c) = \min\{c_i, \gamma\}, \quad i \in N \tag{5}$$

where $\gamma$ is defined as a positive real number. An allocation model using the CEA rule is defined as follows (Equation 6):

$$\sum_{i \in N} CEA_i(N, E, c) = E \tag{6}$$

To apply this in the context of MBC problems, CEA rules must be introduced, as these rules provide an optimal linear programming result. In this case, given a problem (N,E,c) $\in \gamma$ to assign E among claimants according to CEA. The model is solved through a series of linear programming problems, where the states and demands are updated in each iteration until the optimal claim is reached. The general model is presented in Equation (7):

$$\max w^k \quad \text{subject to} \quad A_k x \leq b_k, \ x \geq 0, \tag{7}$$

where:

- $A_k$ is the incidence matrix describing the relationship between the active claimants and the active states.
- $b_k$ is the vector of residual state capacities at stage $k$.

Formally, let $w^{*k}$ be the optimal value of the linear program $(P_k)$, The model is defined in Equation (8):
max $w^{*k}$

$$\text{s.}\alpha \sum_{i\in N} x_i \leq E \tag{8}$$

$$(P_k) \qquad \begin{array}{ll} x_i \leq c_i, & \forall\, i\in N, \\ x_i \geq \sum_{m=1}^{k-1} y^0\left(c_i - \sum_{h=1}^{m-1} w^h\right) w^l + y^0\left(c_i - \sum_{h=1}^{k-1} w^h\right) w^k, & \forall\, i\in N, \\ x_i \geq 0, & \forall\, i\in N, \text{ and } w^k \geq 0. \end{array}$$

Where:

$$d_i^m = \min\left\{c_i - \sum_{h=1}^{m-1} y^0(d_i^h)w^h,\ \min_{j\in\psi(i)}\left\{e_j - \sum_{t\in N;\, j\in\psi(t)} \sum_{r=1}^{m-1} y^0(d_t^r)w^r\right\}\right\} \tag{9}$$

Now, let us see what: If satisfying

$$\sum_{i\in N}\sum_{m=1}^{k} y^0\left(c_i - \sum_{h=1}^{m-1} w^h\right) w^{*m} = E \tag{10}$$

Equation [10] represents the distribution of the total resource E is distributed among the agents i∈N through k progressive resolution stages, which are characteristic of the MBC model.

Then, the CEA allocation must satisfy Equation (11):

$$CEA_i(N,E,c) = \sum_{m=1}^{k} y^0\left(c_i - \sum_{h=1}^{m-1} w^h\right) w^{*m}. \tag{11}$$

It should be noted that $d_i^m$ shows whether the claimant i may continue receiving assignments during the stage m, taking into account what has already been assigned in the previous steps and the remaining availability in the issues in which they are involved (Equation 9).

Once the general linear programming model has been defined $P_k$, We can illustrate the model by showing the first two iterations: (See equation 11).

**The first step** is proposed as follows

$$\begin{array}{ll} \max & w^1 \\ \text{s.}\alpha & \sum_{i\in N} x_i \leq E, \\ (P1) & x_i \leq c_i, \quad \forall\, i\in N, \\ & x_i \geq w^1, \quad \forall\, i\in N, \\ & x_i \geq 0, \quad \forall\, i\in N, \quad w^1 \geq 0. \end{array} \tag{11}$$

Where $w^{*1}$ the solution to the linear problem $(P_1)$. If $n\, w^{*1} = E$,then equation (12) establishes that

$$CEA_i(N,E,c) = w^{*1}. \tag{12}$$

**Step two:** Since the resource is not exhausted in the first allocation, the constraints are updated and $(P_2)$ is described as follows: (See equation 13).

$$
\begin{aligned}
&\max\ w^2 \\
&\text{s.}\alpha \sum_{i \in N} x_i \leq E, \\
&(P2) x_i \leq c_i, \quad \forall i \in N, \\
&\qquad x_i \geq w^{*1} + y^0(c_i - w^{*1})\, w^2, \quad \forall i \in N, \\
&\qquad x_i \geq 0, \quad \forall i \in N, \quad w^2 \geq 0,
\end{aligned}
\tag{13}
$$

where, for each $\in \mathbb{R}$,

$$
y^0(d) = \begin{cases} 0 & \text{if } d \leq 0 \\ 1 & \text{otherwise.} \end{cases}
\tag{14}
$$

The indicator function $y^0(d)$ is defined in equation (14). This function determines whether an agent participates in the allocation process at a given stage. Let $w^{*2}$ be the optimal value of $(P_2)$. If $n\, w^{*2} + \sum_{i \in N} y^0(c_i - w^{*1})\, w^{*2} = E$, then $CEA_i(N, E, c) = w^{*1} + y^0(c_i - w^(*1))w^(*2)$.If there are still sources to be allocated, the procedure continues. In this way, each step k consists of creating a linear problem $(P_k)$ such restrictions are adjusted based on the allocations made in previous iterations. Thus, the optimal solution $x^(*k)$ to the last linear programming problem is as follows: (See Equation 15).

$$
CEA_i(\mathcal{M}, N, E, c, \psi) = \sum_{m=1}^{k} y^0(d_i^m)\, w^{*m}.
\tag{15}
$$

Notably, the CEA rule for bankruptcy problems with cross-claims follows the succession of linear programming problems.

The CEA–MBC procedure thus comprises a sequence of linear programming problems, one per allocation stage. At least one state becomes exhausted, or one claimant exits the process, at each iteration. Therefore, the number of iterations is bounded by $|\mathcal{M}| + |N|$. Because each stage solves a linear program with a single decision variable and linear constraints, the proposed method is computationally efficient and scalable for practical logistics applications. The CEA rule algorithm for MBC is proposed below.

**Algorithm 1. Iterative CEA–MBC Allocation Procedure**

Given an MBC problem $(\mathcal{M}, N, E, c, \psi)$, where:

- $\mathcal{M}$ is the set of states,
- $N$ is the set of claimants,
- $E = (e_i)_{i \in \mathcal{M}}$ are state capacities,
- $c = (c_j)_{j \in N}$ are claims,
- $\psi(j) \subseteq \mathcal{M}$ is the set of states linked to claimant $j$.

**Initialization ($k = 0$)**

$$
e_i^{(0)} = e_i, \quad i \in \mathcal{M}, \tag{16}
$$

$$
x_j^{(0)} = 0, \quad j \in N, \tag{17}
$$

$$
\mathcal{M}^{(0)} = \mathcal{M}, \quad N^{(0)} = N. \tag{18}
$$

**Iterative Step** ($k = 1, 2, \ldots$)

*Step 1. Compute feasible equal increment.* Determine the optimal step value

$$w_k = \max \left\{ w \geq 0 : \sum_{\substack{j \in N^{(k-1)}: \\ i \in \psi(j)}} w \leq e_i^{(k-1)},\ \forall i \in \mathcal{M}^{(k-1)},\quad x_j^{(k-1)} + w \leq c_j,\ \forall j \in N^{(k-1)} \right\}. \tag{19}$$

*Step 2. Allocate to the active claimants.*

$$x_j^{(k)} = \begin{cases} x_j^{(k-1)} + w_k, & j \in N^{(k-1)} \\ x_j^{(k-1)}, & \text{otherwise.} \end{cases} \tag{20}$$

*Step 3. Update the state capacities.*

$$e_i^{(k)} = e_i^{(k-1)} - \sum_{\substack{j \in N^{(k-1)}: \\ i \in \psi(j)}} w_k, \quad i \in \mathcal{M}^{(k-1)}. \tag{21}$$

*Step 4. Update the active sets.* Remove the exhausted states:

$$\mathcal{M}^{(k)} = \{ i \in \mathcal{M}^{(k-1)} : e_i^{(k)} > 0 \}. \tag{22}$$

Remove claimants who reached their demand or are linked to at least one exhausted state:

$$N^{(k)} = \{ j \in N^{(k-1)} : x_j^{(k)} < c_j \text{ and } \psi(j) \subseteq \mathcal{M}^{(k)} \}. \tag{23}$$

**Stopping rule.** Stop if

$$\mathcal{M}^{(k)} = \varnothing \quad \text{or} \quad N^{(k)} = \varnothing \quad \text{or} \quad w_k = 0. \tag{24}$$

**Output.**

$$x = (x_j^{(k)})_{j \in N}. \tag{25}$$

Clarifying+ the operational meaning of each stage: Stage 1 determines the maximum allocation amount without violating any capacity or individual demand constraints. Stage 2 applies this increase to all active producers, preserving the principle of equal treatment. Stage 3 updates municipal capacities after the allocation and stage 4 removes depleted states and their associated producers, ensuring the structural consistency of the next iteration.

## 5. Properties

In the context of bankruptcy problems with multiple states and cross-claims, a series of properties are defined that impose conditions that each distribution rule must meet. The proposed CEA–MBC rule inherits the main axiomatic properties established for MBC allocation rules, including efficiency, Pareto efficiency, equal treatment of equals, claims truncation invariance, respect for minimal rights, conditional equal division, securement, conditional full compensation, claim monotonicity, and Lorenz dominance. Formal definitions and characterizations of these properties are provided in Supplementary Material and in Acosta-Vega et al. (2022, 2023, 2025).

## 6. Results and Discussion

### 6.1 Case Study

Below is an applied case study, for which the department of Magdalena, Colombia, was chosen as the location for its development. Minsalud (2024) mentioned that the department has a monetary poverty rate of 61.1% of its inhabitants, and extreme monetary poverty stands at 24.4%. However, with regard to international trade, (Ministry of Commerce, Industry, and Tourism – ProColombia, 2025) highlights that, thanks to the efforts of the Ministry of Commerce, Industry, and Tourism, the department's presence on the international stage has been fundamental in strengthening its exports at the national level. Companies such as Grupo Daabon, Red Ecolsierra, and Gradesa participated in strategic platforms such as Macrorrueda de Negocios 105, Expo Osaka, and Showroom Aruba–Curaçao, which allowed them to expand their commercial opportunities and consolidate relationships with buyers from Asia and the Dutch Caribbean.

On the other hand, MinCIT (2026) cited DANE (2025) mentions that in 2025, the department's non-mining and energy exports were valued at \$1.168 billion in 2025, representing a 3.72% share at the national level, while the agro-industrial sector had a 48.1% share of exports at the departmental level. Additional socioeconomic and export indicators for the study region are reported in Supplementary Table S2. This demonstrates that the department is a key link in the country's productive growth and that it is necessary to implement cooperation strategies that allow the actors in the territory's logistics chain to coordinate with the aim of establishing a fair and equitable distribution of the profits obtained by these companies to small producers in the department. For this reason, we focus on various associations of small fruit producers of products such as mango, guava, papaya, melon, and pineapple that have organized themselves to sell directly in regional and national markets, with a view to exporting. However, they are constrained by limited logistics infrastructure to transport their produce from rural villages and districts to the ports of Santa Marta and Barranquilla. A rural fruit logistics cooperative, supported by an inter-institutional program (with the participation of municipal councils, universities and the departmental government), has a limited daily capacity (in tonnes) to transport this production. Each group of producers is entitled to claim part of this capacity based on their projected volume, previous involvement with the cooperative, and compliance with health and trade requirements. Given that total claims exceed the available capacity in each logistics zone, fair distribution mechanisms based on the criteria of equity, efficiency, and rural cooperation must be applied. The complete case-study dataset, including state capacities, claimant characteristics, commodity types, locations, and cross-state claims, is reported in Supplementary Table S3. The corresponding claimant–state interaction structure and logistics network representation are provided in Supplementary Figure S1. The above situation can be represented as a bankruptcy problem with multiple states and cross-claims as follows figure 2.

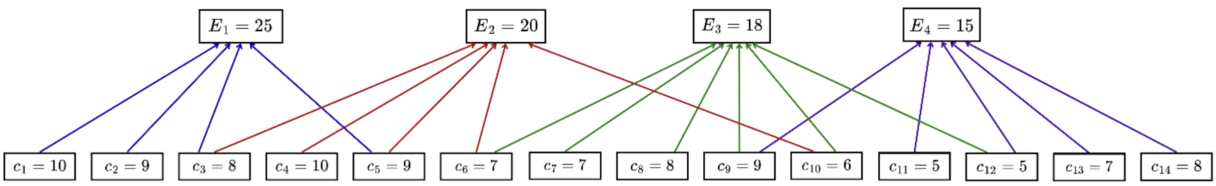


**Figure 2** Hierarchical structure of the case study

Consider the above case as a bankruptcy problem with multiple states and cross-claims $MBC = (\mathcal{M}, N, E, c, \psi)$, defined by

- $\mathcal{M} = \{1, 2, 3, 4\}$,
- $N = \{1, 2, 3, \ldots, 14\}$; $E = (25, 20, 18, 15)$,
- $c = (10, 9, 8, 10, 9, 7, 7, 8, 9, 6, 5, 5, 7, 8)$,

Furthermore,

$$\psi(1) = \{1\},\ \psi(2) = \{1\},\ \psi(3) = \{1,2\},\ \psi(4) = \{2\},\ \psi(5) = \{1,2\},\ \psi(6) = \{2,3\},\ \psi(7) = \{3\},$$
$$\psi(8) = \{3\},\ \psi(9) = \{3,4\},\ \psi(10) = \{2,3\},\ \psi(11) = \{4\},\ \psi(12) = \{3,4\},\ \psi(13) = \{4\},\ \psi(14) = \{4\}.$$

The optimal value of $(P_1)$ is $w^{*1} = 3$, an amount allocated to every fruit grower. The resources of the states associated with Ciénaga and Sitionuevo are allocated in their entirety in this stage. Therefore, fruit growers 6, 7, 8, 9, 10, 11, 12, 13, and 14, who hold rights to these states, no longer participate in the next step of the distribution process; in this step, they are guaranteed everything they have obtained up to the previous step. The optimal value of $(P_2)$ is $w^{*2} = 1.67$ (Ton), and the resources of the state associated with Aracataca are allocated in their entirety. Consequently, fruit growers 3, 4, and 5, who hold rights to it, no longer participate in the next step of the distribution process, while being guaranteed everything they have obtained up to that point. The optimal value of $(P_3)$ is $w^{*3} = 3.17$ (Ton), and the resources of the state associated with Fundación are allocated in their entirety; therefore, fruit growers 1 and 2, who hold rights to it, no longer participate, having already received the guarantee of their accumulated allocation. The CEA–MBC rule is thus computed through three allocation stages. Detailed stage-by-stage allocations are reported in Table 1, while the main allocation outcomes are discussed below.

**Table 1** Resulting allocations by stages of the proposed mechanism under constrained capacity

| Claimant | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P_1$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 42 |
| $P_2$ | 1.67 | 1.67 | 1.67 | 1.67 | 1.67 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8.33 |
| $P_3$ | 3.17 | 3.17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6.33 |
| Total | 7.83 | 7.83 | 5 | 4.67 | 4.67 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 56.67 |

In logistical terms, it is important to differentiate between what producers receive and each municipality's capacity. The total amount allocated to producers is 56.67 tonnes, while the municipalities have a total handling capacity of 78 tonnes (25 + 20 + 18 + 15). These two figures represent different operational perspectives and should not be compared directly. Distribution decisions are made per producer in the multiissue allocation structure, using equal allocations at each stage. However, capacity constraints are managed at the municipal level. Because some producers are linked to more than one municipality, a single allocation to a producer may simultaneously useundefined capacity in several locations. Therefore, the total amount delivered to producers does not match the sum of the municipal capacities. Therefore, operational consistency must be verified by determining whether each municipality's capacity is being fully utilized. Table 6 shows that the available capacity in each municipality is completely exhausted by the end of Stage 3. This confirms that the allocation process is operationally consistent and no municipal handling capacity remains unused. Detailed state-wise capacity consumption and residual capacities after each allocation stage are reported in Table 2.

**Table 2** State-wise capacity consumption and residual capacity by stage (s)

| State | Initial Capacity | Stage 1 Consump-tion | Residual after S1 | Stage 2 Consump-tion | Residual after S2 | Stage 3 Consump-tion |
|---|---|---|---|---|---|---|
| $E_1$ | 25 | 12 | 13 | 6.67 | 6.33 | 6.33 |
| $E_2$ | 20 | 15 | 5 | 5 | 0 | 0 |
| $E_3$ | 18 | 18 | 0 | 0 | 0 | 0 |
| $E_4$ | 15 | 15 | 0 | 0 | 0 | 0 |

**Remark.** Claimant-level totals (56.67 tons) and aggregate state capacities (78 tons) represent different accounting perspectives. A single allocation contributes simultaneously to multiple-issue capacity constraints due to cross-claims. Therefore, full consistency is verified through statewise exhaustion.

The application of the fair distribution model in the logistics field allowed us to demonstrate the effectiveness of the multi-issue bankruptcy (MBC) approach with cross-claims for allocating scarce resources among different agents in a supply chain. In this context, the CEA Rule provided a fair and transparent criterion for distributing the logistics capacities of each producer in the case study, respecting each resource's limitations and ensuring a fair allocation. The results revealed that the procedure achieves a reasonable balance between efficiency and fairness, avoiding overallocation of resources and providing guarantees for cooperation between the actors involved.

From a theoretical perspective, the model shows that the extension of the CEA rule to environments with multiple interrelated resources maintains key properties such as consistency, conditional equality, and Pareto efficiency (Acosta-Vega et al., 2022). This allows solutions to remain consistent despite changes in the set of agents or resources, giving greater relevance to the integrity of the alliances between them. Thus, the formulation applied in logistics represents a generalization consistent with the principles expressed within the field of cooperative game theory, providing a conceptual basis for decision-making.

### 6.2 Benchmark Proportional–MBC

To strengthen the empirical support of the proposed allocation mechanism, the CEA–MBC outcome was compared with the Proportional rule adapted to multistate problems with cross-claims. The proportional benchmark follows the proportionality-based formulation proposed by Acosta-Vega et al. (2023) for crossed-claim multi-issue settings.

Under the Proportional–MBC rule, each claimant receives an allocation proportional to their claim, adjusted through the multistate cross-claim structure. In contrast, the CEA–MBC procedure assigns equal feasible increments at each stage and updates the active sets whenever a state is exhausted or a claimant is affected by the exhaustion of any state in $\psi(j)$. Consequently, CEA–MBC tends to limit the dominance of large claims and preserves stagewise equal treatment, whereas Proportional–MBC reproduces proportional fairness with respect to declared demands.

Supplementary Table S4. Reports the proportional MBC allocation by stage for the case study, allowing a direct comparison with the CEA MBC allocation presented in Table 1. The benchmark highlights that both approaches satisfy feasibility, but they differ in distributive profile: proportionality emphasizes claim-weighted shares, whereas CEA – MBC enforces equal incremental treatment under multistate capacity constraints.

From a distributional perspective, the Proportional–CBM rule allocates resources in direct proportion to declared demands. Therefore, claimants with higher demands receive larger shares at each stage. This preserves proportional equity but can amplify structural differences when demand heterogeneity is significant.

Conversely, the CEA–CBM distributes equal feasible allocations at each stage, regardless of the initial demand size. This equitable treatment across stages limits the dominance of large claimants and promotes greater horizontal equity among producers operating under shared logistical constraints. Small- and medium-sized producers benefit from the incremental structure before larger demands accumulate disproportionate allocations.

From an efficiency perspective, both mechanisms satisfy feasibility: all state capacities are exhausted by the end of the allocation process. However, the mechanisms differ in their distributional efficiency profile. The proportional rule prioritizes consistency in the weighting of demands, while the CEA-CBM rule prioritizes equitable incremental access to limited infrastructure.

In logistic terms, the proportional approach reflects demand-weighted capacity access, whereas the CEA–CBM approach imposes uniform access increments under multiissue constraints. Therefore, the choice between the two depends on whether the policy objective prioritizes proportional ownership or incremental equity under shared infrastructure scarcity conditions.

### 6.3 CEL-Inspired Benchmark Adaptation

To broaden the comparative analysis, we incorporate a simplified benchmark inspired by the Constrained Equal Losses (CEL) rule. In classical bankruptcy problems, CEL is based on the principle that losses should be equalized across claimants as much as possible. Formally, given claims $d_i$and an available estate $E$, the rule determines allocations of the form.

$$x_i = \max\{d_i - \lambda, 0\}, \tag{23}$$

where $\lambda$ is a common loss level chosen such that the aggregate allocation equals the available estate.

This equal-losses principle provides a natural normative counterpart to the CEA–MBC rule. However, a full theoretical extension of CEL to multi-issue bankruptcy problems with cross-claims remains open and lies beyond the scope of this study; therefore, a complete CEL–MBC formulation is not claimed here. Instead, a simplified CEL-inspired benchmark adaptation is introduced for comparative purposes.

Let $d_i$ denote the claim of claimant $i$, and let $x_i$ be the allocation assigned to claimant $i$. The corresponding loss is defined as

$$\ell_i = d_i - x_i. \tag{24}$$

Claimant allocations must satisfy both individual claim bounds and state-level feasibility constraints in this MBC environment. Thus, the adapted benchmark is defined over the feasible region

$$0 \leq x_i \leq d_i, \quad \forall i \in N, \tag{25}$$

$$\sum_{i:\, k \in S_i} x_i \leq e_k, \quad \forall k \in E, \tag{26}$$

where $S_i$ denotes the set of states associated with claimant $i$, and $e_k$ is the available capacity of state $k$. Within this feasible set, the CEL-inspired benchmark follows the equal-losses rationale by seeking allocations that make the claimant losses as similar as possible while preserving consistency with the problem's multi-state structure. The benchmark may be interpreted as searching for a feasible allocation vector such that.

$$\ell_i \approx \lambda, \quad \forall i \in N, \tag{27}$$

for some common loss level $\lambda$, subject to the MBC setting's feasibility requirements.

This benchmark preserves three essential features for this comparison. First, it is based on the claim structure induced by the MBC environment. Second, it remains compatible with the problem's state-capacity constraints. Third, it provides a loss-based counterpart to the proposed CEA–MBC rule, thereby allowing a more informative comparison between alternative normative allocation principles. The adapted CEL benchmark is obtained in a single feasible step in the case study considered here. More complex instances may require iterative adjustments, which are left for future research. To further assess the distributive behavior of the proposed framework, a comparison with a CEL-inspired benchmark was conducted. The complete claimant-level allocations obtained under this benchmark are reported in Supplementary Table S5. Table 1 presents the claimant-level allocations obtained under CEA-MBC, Proportional-MBC, and the CEL–inspired benchmark for the case study.

**Table 3** Claimant-level allocations under the three allocation rules

| Claimant | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CEA–MBC | 7.83 | 7.83 | 4.67 | 4.67 | 4.67 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 56.67 |
| Proportional–MBC | 8.38 | 7.54 | 4.28 | 5.34 | 4.81 | 3 | 3 | 3.43 | 3.86 | 2.57 | 2.25 | 2.14 | 3.15 | 3.60 | 57.34 |
| CEL-inspired benchmark | 6 | 5 | 4 | 6 | 5 | 3 | 3 | 4 | 5 | 2 | 1 | 1 | 3 | 4 | 52.00 |

**Note:** The table reports the corresponding final allocation obtained for each claimant under CEA–MBC, Proportional–MBC, and the CEL-inspired benchmark. The last column shows the total allocation delivered by each rule.

Table 3 presents the claimant-level allocations obtained under the three allocation rules. Proportional–MBC yields the highest total allocation, followed closely by CEA–MBC, whereas the CEL-inspired benchmark produces a lower total allocation. However, the CEL-inspired rule reflects a more balanced loss structure across claimants because it is based on a common-loss rationale, making it a useful loss-based reference for assessing the proposed CEA–MBC rule's distributive behavior.

**Table 4** Comparative performance of allocation rules

| Rule | Total allocated | Unmet demand | Average satisfaction rate | Jain's Fairness Index | Fully used states |
|---|---|---|---|---|---|
| CEA–MBC | 56.67 | 51.33 | 0.5208 | 0.9237 | 4* |
| Proportional–MBC | 57.34 | 50.66 | 0.5143 | 0.9327 | 4* |
| CEL-inspired benchmark | 52.00 | 56.00 | 0.4561 | 0.9282 | 2 |

**Note:** Full use is observed for CEA–MBC and Proportional–MBC up to rounding differences in the reported claimant-level allocations.

Table 4 summarizes the performance of the three allocation rules. Proportional–MBC achieves the highest total allocation (57.34) and the lowest unmet demand (50.66), indicating that this rule makes the greatest use of the available capacity in aggregate terms. CEA–MBC yields a slightly lower total allocation (56.67) but attains the highest average satisfaction ratio (0.5208), suggesting a stronger average claimant-level coverage. In contrast, the CEL–inspired benchmark produces the lowest total allocation (52.00) and the highest unmet demand (56.00), which is consistent with its loss-balancing rationale. In fairness terms, all three rules display high values of Jain's index when over claimant-level satisfaction ratios. Among them, Proportional-MBC attains the highest fairness value (0.9327), followed by the CEL–inspired benchmark (0.9282) and CEA–MBC (0.9237). Although the differences are not large, the results suggest that the three approaches generate balanced distributions in this case study, with the proportional benchmark showing a slight advantage under this particular metric.

However, a relevant distinction emerges from the normative logic underlying each rule. While proportional-MBC favors an allocation pattern more closely aligned with claim magnitudes and CEA-MBC is based on an equal-awards rationale, the CEL-inspired benchmark reflects a loss-oriented perspective by imposing a common loss level whenever feasibility allows. Even though the CEL–inspired benchmark allocates fewer total resources, it provides a meaningful comparative reference because it captures a different distributive principle. This broader comparison strengthens the empirical assessment of the proposed rule by showing how its allocation pattern differs from a proportional benchmark and an alternative loss-based approach.

### 6.4 Alternative Claimant–State Structures for Robustness Analysis

To complement the real case study, two scenarios were constructed by replicating the hierarchical logic of the original MBC problem on a smaller scale, using 8 claimants and 3 states. The claimant–state connectivity structure of the scenarios differs, representing settings with fewer and more crossings between claimants and states; this design allows the allocation rules' behavior to be examined under alternative crossed-claim configurations. The first scenario represents a lower-connectivity setting, whereas the second represents a higher-connectivity setting. The state capacities are fixed as follows: $(E_1, E_2, E_3)$=(25,20,18) for the lower-connectivity scenario, and $(E_1, E_2, E_3)$=(25,20,18) for the higher-connectivity scenario. Figures 3 and 4 show the corresponding hierarchical representations. The complete results for the lower claimant–state connectivity scenario are provided in Supplementary Table S6.

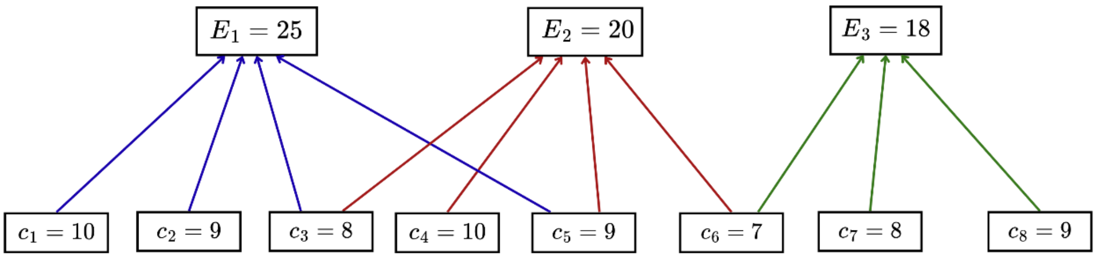


**Figure 3** Hierarchical representation of the scenario with lower claimant–state connectivity

To assess the effect of denser cross-claim structures, additional experiments were conducted under a higher claimant–state connectivity configuration. Detailed allocation results are reported in Supplementary Table S7.

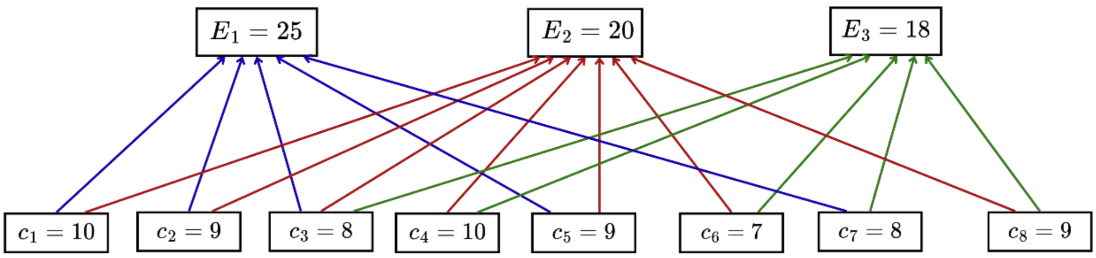


**Figure 4** Hierarchical representation of the scenario with higher claimant–state connectivity

Additional robustness analyses under alternative claimant–state structures were conducted to evaluate the sensitivity of the proposed allocation mechanism to different network configurations. Detailed allocation results and performance indicators are reported in Supplementary Table S8.

Under the lower-connectivity structure, CEA-MBC achieves the highest total allocation (51.00) and the lowest unmet demand (19.00), whereas Proportional-MBC and the CEL-inspired benchmark yield lower aggregate allocations. All three rules perform well in terms of fairness, although the CEL–inspired benchmark attains the highest Jain's index (0.9937), reflecting its loss-balancing rationale. Under the higher-connectivity structure, CEA-MBC again yields the largest total allocation (26.59) and the lowest unmet demand (43.41), whereas Proportional-MBC and the CEL-inspired benchmark produce lower allocations. However, Proportional–MBC reaches the highest fairness level in this case, with a Jain's index of 0.9998, indicating an almost perfectly uniform relative satisfaction pattern. Overall, the results reveal a clear trade-off between the aggregate allocation performance and the distributive uniformity across the alternative structures and rules.

## 7. Conclusions

This article has shown that multistate bankruptcy problems with cross-claims provide a useful framework for addressing resource allocation challenges in interdependent logistics settings. In contexts where actors share critical infrastructure, transportation assets, or operational capacities, the design of fair allocation mechanisms is essential for preserving cooperation and operational continuity. The analysis confirms that allocation rules, and in particular the constrained equal awards rule (CEA), can generate feasible and practically meaningful solutions, contributing to cooperative arrangements' stability and the effective use of available resources. The results indicate that this approach can capture the operational constraints that characterize logistics systems while incorporating the multistate nature of distributive conflicts. The proposed formulation makes it possible to represent the interdependence of claims, the system's limited capacity, and the need to balance efficiency and fairness in the assignment of scarce resources. This study illustrates how concepts originally developed in the bankruptcy literature can be successfully adapted to practical decision-making problems in supply chains and logistics management. The comparative analysis with Proportional-MBC and the additional CEL-inspired benchmark further strengthens the paper's contribution by showing that the proposed framework can be examined not only from an equal-awards perspective but also against proportional and loss-based allocation principles. Likewise, the inclusion of alternative claimant – state structures provides additional evidence that the proposed rule's behavior can be assessed beyond a single real case study.

This study also opens several directions for future research. In particular, extending the comparative analysis to other allocation rules, such as random arrival, and examining their performance in different logistics settings, including agricultural and industrial supply chains in which actors share strategic resources, such as transportation networks, terminals, or storage facilities, would be valuable. Further research may also deepen the structural analysis of alternative claimant – state configurations and more complex interdependence patterns. A broader quantitative validation, including additional fairness measures, envy-freeness proxies, and systematic sensitivity analysis under alternative crossclaim structures, remains a relevant avenue for future work. Such extensions require additional computational experimentation and structural modeling beyond the scope of this applied study but constitute a natural continuation of the framework developed here. Overall, the findings suggest that multiissue bankruptcy problems with cross-claims offer a robust and flexible basis for designing logistics allocation mechanisms. More broadly, they support the development of supply-chain coordination schemes that are not only operationally feasible but also more equitable and responsive to resource scarcity conditions.

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## Author Contributions

Conceptualization, R.A-V.; Methodology, R.A-V., J.B-C. and S.A-C.; Formal analysis, R.A-V., J.B-C., S.A-C. and M.J.C; Software, M.J.C. and E.J.D-D.; Validation, R.A-V. and M.J.C.; Investigation, R.A-V., M.J.C., J.B-C. and S.A-C.; Writing—original draft preparation, R.A-V. and S.A-C.; Writing—review and editing, R.A-V., M.J.C. and E.J.D-D.

## Conflict of Interest

The authors declare that there is no conflict of interest.

**References**


Acosta-Vega, R. K., Algaba, E., & Sánchez-Soriano, J. (2022). Multi-issue bankruptcy problems with crossed claims. *Annals of Operations Research*, *318*, 749–772. https://doi.org/10.1007/s10479-021-04470-w

Acosta-Vega, R. K., Algaba, E., & Sánchez-Soriano, J. (2023). Design of water quality policies based on proportionality in multi-issue problems with crossed claims. *European Journal of Operational Research*, *311*(2), 777–788.

Acosta-Vega, R. K., Algaba, E., & Sánchez-Soriano, J. (2025). On priority in multi-issue bankruptcy problems with crossed claims. *Mathematics*, *13*(2), 246. https://doi.org/10.3390/math13020246

Algaba, E., Fragnelli, V., & Sánchez-Soriano, J. (Eds.). (2019). *Handbook of the shapley value*. CRC Press. https://doi.org/10.1201/9781351241410

Algaba, E., Márquez, G., Martínez-Lozano, J., & Sánchez-Soriano, J. (2023). A novel methodology for public management of annual greenhouse gas emissions in the european union. *Socio-Economic Planning Sciences*, *89*, 101697. https://doi.org/10.1016/j.seps.2023.101697

Audy, J.-F., D'Amours, S., & Rousseau, L.-M. (2010). Cost allocation in the establishment of a collaborative transportation agreement—an application in the furniture industry. *Journal of the Operational Research Society*, *62*(6), 960–970. https://doi.org/10.1057/jors.2010.53

Aumann, R. J., & Maschler, M. (1985). Game theoretic analysis of a bankruptcy problem from the talmud. *Journal of Economic Theory*, *36*(2), 195–213. https://doi.org/10.1016/0022-0531(85)90102-4

Bergantiños, G., Chamorro, J. M., Lorenzo, L., & Lorenzo-Freire, S. (2018). Mixed rules in multi-issue allocation situations. *Naval Research Logistics*, *65*, 66–77. https://doi.org/10.1002/nav.21785

Bergantiños, G., Gómez-Rúa, M., Llorca, N., Pulido, M., & Sánchez-Soriano, J. (2020). Allocating costs in set covering problems. *European Journal of Operational Research*, *284*, 1074–1087. https://doi.org/10.1016/j.ejor.2020.01.031

Bergantiños, G., Lorenzo, L., & Lorenzo-Freire, S. (2009). A characterization of the proportional rule in multi-issue allocation situations. *Operations Research Letters*, *38*(1), 17–19. https://doi.org/10.1016/j.orl.2009.10.003

Bergantiños, G., Lorenzo, L., & Lorenzo-Freire, S. (2011). New characterizations of the constrained equal awards rule in multi-issue allocation situations. *Mathematical Methods of Operations Research*, *74*(3), 311–325. https://doi.org/10.1007/s00186-011-0364-3

Calleja, P., Borm, P., & Hendrickx, R. (2005). Multi-issue allocation situations. *European Journal of Operational Research*, *164*(3), 730–747. https://doi.org/10.1016/j.ejor.2003.10.042

Dinar, A., & Hogarth, M. (2015). Game theory and water resources: Critical review of its contributions, progress and remaining challenges. *Foundations and Trends in Microeconomics*, *11*(1–2), 1–139. https://doi.org/10.1561/0700000066

Duro, J. A., Giménez-Gómez, J. M., & Vilella, C. (2020). The allocation of CO2 emissions as a claims problem. *Energy Economics*, *86*, 104652. https://doi.org/10.1016/j.eneco.2019.104652

Frisk, M., Göthe-Lundgren, M., Jörnsten, K., & Rönnqvist, M. (2006). *Cost allocation in collaborative forest transportation* (No. 2006/15). NHH Department of Finance and Management Science. https://doi.org/10.2139/ssrn.969362

Gallastegui, M. C., Iñarra, E., & Prellezo, R. (2002). Bankruptcy of fishing resources: The northern european anglerfish fishery. *Marine Resource Economics*, *17*, 291–307. https://doi.org/10.1086/mre.17.4.42629371

García-Jurado, I. (2005). Some comments on the fundamentals and the applications of the shapley value.

Giménez-Gómez, J. M., Teixidó-Figueras, J., & Vilella, C. (2016). The global carbon budget: A conflicting claims problem. *Climatic Change*, *136*, 693–703. https://doi.org/10.1007/s10584-016-1633-1

González Rodríguez, P. (2002). Aspectos económicos de la provisión pública de servicios sanitarios.

Gozálvez, J., Lucas-Estañ, M. C., & Sánchez-Soriano, J. (2012). Joint radio resource management for heterogeneous wireless systems. *Wireless Networks*, *18*, 443–455. https://doi.org/10.1007/s11276-011-0410-3

Gutiérrez, E., Llorca, N., Sánchez-Soriano, J., & Mosquera, M. (2018). Sustainable allocation of greenhouse gas emission permits for firms with leontief technologies. *European Journal of Operational Research*, *269*(1), 5–15. https://doi.org/10.1016/j.ejor.2017.10.011

Izquierdo, J. M., & Timoner, P. (2016). *Constrained multi-issue rationing problems* (UB Economics Working Papers No. 2016/347). University of Barcelona School of Economics. https://doi.org/10.2139/ssrn.2841642

Karagök, Y. (2006). Methoden zur berechnung des nukleolus kooperativer spiele mit einer anwendung für die schweiz.

Lemaire, J. (1984). An application of game theory: Cost allocation. *ASTIN Bulletin*, *14*(1), 61–81. https://doi.org/10.1017/S0515036100004815

Liu, L., & Liao, W. (2021). Optimization and profit distribution in a two-echelon collaborative waste collection routing problem from economic and environmental perspective. *Waste Management*, *120*, 400–414. https://doi.org/10.1016/j.wasman.2020.09.045

Lorenzo-Freire, S., Casas-Méndez, B., & Hendrickx, R. (2010). The two-stage constrained equal awards and losses rules for multi-issue allocation situations. *Top*, *18*(2), 465–480. https://doi.org/10.1007/s11750-009-0073-8

Lucas-Estañ, M. C., Gozálvez, J., & Sánchez-Soriano, J. (2012). Bankruptcy-based radio resource management for multimedia mobile networks. *Transactions on Emerging Telecommunications Technologies*, *23*, 186–201. https://doi.org/10.1002/ett.1525

Maflahah, I., Wirjodirdjo, B., & Karningsih, P. D. (2024). Improving salt farmer's bargain power through demand allocation and profit sharing: A cooperative game approach. *International Journal of Technology*, *15*(1), 110–120.

Ministry of Commerce, Industry, and Tourism – ProColombia. (2025, December 9). *Magdalena promotes its international profile in tourism, exports, and investment*. https://procolombia.co/sala-de-prensa/noticias/magdalena-impulsa-su-proyeccion-internacional-en-turismo-exportaciones-e-inversion

Moreno-Ternero, J. D. (2009). Proportional rule for multi-issue bankruptcy problems. *Economics Bulletin*, *29*(1), 483–490.

Niyato, D., & Hossain, E. (2006). A cooperative game framework for bandwidth allocation in 4G heterogeneous wireless networks. *Proceedings of IEEE ICC 2006*. https://doi.org/10.1109/ICC.2006.255766

O'Neill, B. (1982). A problem of rights arbitration from the talmud. *Mathematical Social Sciences*, *2*(4), 345–371. https://doi.org/10.1016/0165-4896(82)90029-4

Pulido, M., Sánchez-Soriano, J., & Llorca, N. (2002). Game theory techniques for university management: An extended bankruptcy model. *Annals of Operations Research*, *109*(1/4), 129–142. https://doi.org/10.1023/a:1016395917734

Sánchez-Soriano, J., Llorca, N., & Algaba, E. (2016). An approach from bankruptcy rules applied to the apportionment problem in proportional electoral systems. *Operations Research and Decisions*, *26*, 127–145. https://doi.org/10.5277/ord160208

Singh, C., & Altman, E. (2011). The wireless multicast coalition game and the non-cooperative association problem. *2011 Proceedings IEEE INFOCOM*, 2705–2713.

Thomson, W. (2003). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: A survey. *Mathematical Social Sciences*, *45*(3), 249–297. https://doi.org/10.1016/s0165-4896(02)00070-7

Thomson, W. (2015). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: An update. *Mathematical Social Sciences*, *74*, 41–59. https://doi.org/10.1016/j.mathsocsci.2014.09.002

Thomson, W. (2019). *How to divide when there isn't enough*. Cambridge University Press. https://doi.org/10.1017/9781108161107

Wickramage, H. M., Roberts, D. C., & Hearne, R. R. (2020). Water allocation using the bankruptcy model: A case study of the missouri river. *Water*, *12*(3), 619. https://doi.org/10.3390/w12030619

Young, H. P. (1994). *Equity, in theory and practice*. Princeton University Press.

Zheng, X. X., Liu, Z., Li, K. W., Huang, J., & Chen, J. (2019). Cooperative game approaches to coordinating a three-echelon closed-loop supply chain with fairness concerns. *International Journal of Production Economics*, *212*, 92–110. https://doi.org/10.1016/j.ijpe.2019.01.011