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ANALYTIC HIERARCHY PROCESS (AHP) PAIRWISE MATRIX WITH ONE MISSING VALUE

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ABSTRACT

In order to obtain the results of an Analytic Hierarchy Process (AHP), all of the lower or upper triangle elements of the pairwise matrix need to be filled in. As the number of criteria of an AHP increases, the number of elements of the pairwise matrix increases quadratically. This forces an expert to answer a large number of comparisons. This paper studies and analyzes the characteristics of a pairwise matrix when one of its elements is not available. This is one of the efforts to reduce the number of comparisons that need to be provided by an expert. The results show that a complete pairwise matrix that is consistent tends to have the same characteristics (priority sequence and consistency index) as when it has one missing value. Further research is needed so that the number of comparisons can be decreased while still keeping the pairwise matrix consistent.

Keywords: Analytic hierarchy process; Consistency index; Missing value; Pairwise matrix

1. INTRODUCTION

The Analytic Hierarchy Process (AHP) is a Multi-Criteria Decision Making System that was developed by Thomas L. Saaty in the 1970s (Saaty, 1987). It is particularly useful as a method when making decisions on issues that involve multiple criteria and multiple alternatives (Basak & Saaty, 1993; Zhu et al., 2016; Ho & Ma, 2017). Forman (1990) stated that the AHP is flexible, logical and intuitive, and easy to use. Therefore, its use has grown rapidly in a variety of complex decision-making processes (Forman, 1993; Acharya et al., 2017).

The decision makers are required to set up the criteria and sub-criteria and then assign alternatives in a hierarchical form. Afterwards, pairwise comparisons are performed for the criteria and sub-criteria in order to generate the priority of the alternatives. The number of pairwise comparisons required in real-world problems can often be very large (Harker, 1987a; Harker, 1987b). Moreover, decision makers often have different perspectives and influence the results differently (Ivanco et al., 2017). The efficiency of the AHP can be enhanced if the user is able to create a priority without having to complete the entire pairwise comparison (Wedley, 1993; Oliva et al., 2017).

In general, much research has been carried out to reduce the number of pairwise comparisons (Lambert, 1991; Shen et al., 1992). But this research has generally not been conducted in relation to the AHP. Following Wedley (1993) and Bozóki et al. (2016), this study aims to examine the impacts of a pairwise comparison matrix that has not been filled out completely.

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Specifically, the current paper analyzes the consistency of a pairwise comparison matrix when it is complete and when it is incomplete.

2. METHODS

The current study aims to analyze the effects when a researcher does not fill all of the pairwise comparisons in an AHP study. In performing the study, pairwise comparison matrices with five criteria are generated randomly. A total of 1000 pairwise comparison matrices are generated randomly, but only 31 consistent matrices can be used for the experiments. Excel VBA was used to generate the matrices.

Then, for each consistent matrix, one matrix entry was erased randomly. Then, the largest and the smallest weight values, the maximum gap, and the consistency ratios before and after the erase were recorded. For each consistent matrix, 1000 replications of the erasing of one entry of the matrix were performed.

3. RESULTS AND DISCUSSION

The implementation of the research methodology and the experiments underwent verification and validation steps. All of the formula in Excel VBA were checked. The results of the consistency ratio of the Excel VBA were compared by manual calculation.

The characteristics of the 1000 randomly generated pairwise matrices are shown in Table 1. It can be seen that only 31 matrices are consistent, i.e., they have a consistency ratio below 0.1. For each pairwise comparison matrix, one of the pairwise comparisons is erased, with a random selection of the entry to be erased. Table 2 presents the matrix element to be erased.

Table 3 presents a comparison of the consistency ratios before and after the value of the pairwise matrix element is removed. In general, the more consistent a pairwise matrix, the greater the tendency for it to retain its consistency, even when one of the elements is missing. This can be seen in the fact that when the initial consistency ratio is below 0.05, the pairwise matrix remains consistent when one of the elements is missing. In addition, when the initial consistency ratios are between 0.05 and 0.075, the retain percentage is 85%, while when the initial consistency ratios are between 0.075 and 0.1, the retain percentage is only 34%.

Table 1 Characteristics of 1000 random pairwise matrices

Consistency ratio	Frequency
0.000–0.024	0
0.025–0.049	4
0.050–0.074	7
0.075–0.099	20
0.100–0.124	29
0.125–0.149	30
More	910
Total	1000

Table 2 Pairwise matrix element being erased

Group	Matrix Element (Row, Column)										CR
	(1,2)	(1,3)	(1,4)	(1,5)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)	
1	4	3	9	8	1	7	8	5	4	3	0.07838
2	9	5	8	8	1	2	4	3	2	3	0.07339
3	2	3	8	9	6	6	6	5	5	2	0.08807
4	3	2	6	8	1	3	6	2	5	7	0.06373
5	1	5	7	3	3	3	7	3	3	1	0.06948
6	3	6	5	3	1	7	4	4	2	1	0.09875
7	3	2	6	8	4	9	5	4	3	2	0.09198
8	1	3	3	5	1	2	5	3	5	3	0.04315
9	2	3	9	9	2	7	9	2	7	2	0.02837
10	7	3	7	9	2	3	3	2	1	2	0.07695
11	7	9	7	8	3	4	4	2	3	3	0.09235
12	5	2	7	9	1	3	7	8	4	3	0.08498
13	2	4	5	7	1	7	8	3	2	1	0.05426
14	2	6	9	9	3	5	5	7	9	1	0.09909
15	3	1	3	7	1	4	8	4	9	8	0.09217
16	1	1	6	7	3	2	4	3	9	2	0.09109
17	1	2	4	4	1	2	5	3	1	2	0.07661
18	1	7	3	8	3	1	8	1	6	7	0.07556
19	1	6	4	8	3	4	7	1	1	8	0.09766
20	3	5	9	9	3	6	7	2	2	5	0.08452
21	2	4	4	5	1	9	8	3	2	1	0.08163
22	5	9	4	9	3	2	4	1	6	5	0.08849
23	3	7	6	8	4	5	4	4	1	1	0.06671
24	3	3	8	4	1	3	6	3	4	1	0.04389
25	7	3	8	9	2	2	4	1	7	4	0.08869
26	2	4	5	8	1	3	9	4	4	3	0.04874
27	2	2	5	3	2	5	4	4	6	3	0.09598
28	2	3	7	6	3	4	9	3	9	1	0.07222
29	3	1	6	7	3	4	9	4	5	1	0.09252
30	1	2	5	9	7	7	9	5	8	3	0.09339
31	2	7	9	9	2	8	9	3	4	3	0.05559

Table 3 Summary of AHP pairwise comparison matrix with one missing element

Initial conditions			Vacated Condition 1 Matrix Pairwise					Group
Interval CR			Final Weights		Consistency Ratio			
Min	-	Max	Unchanged	Changed	CR>0.1	CR<0.1	% Unchanged	
0.00000	-	0.05000	2877	1123	0	4000	100%	8, 9, 24, 26
0.05001	-	0.07500	4196	1722	1082	5918	85%	2, 4, 5, 13, 23, 28, 31
0.07501	-	0.10000	4889	1982	13129	6871	34%	1, 3, 6, 7, 10, 11, 12, 14, 15, 16, 17, 18, 19, 29, 21, 22, 25, 27, 29, 30

The results are encouraging since the consistency ratio tends to remain consistent. For pairwise comparison matrices that have an initial consistency ratio between 0.0 and 0.05, 100% of the matrices remain consistent. For those that have an initial consistency ratio between 0.05 and 0.075, 85% of the matrices remain consistent. As for the matrices that have an initial consistency ratio between 0.075 and 0.1 (indicating that the matrices are close to the border of consistent), 34% of the matrices remain consistent.

This is expected since only one element of the matrix is missing while the other elements of the consistency matrix remain the same. In this way, future research can be explored to find a method to approximate the value of the missing element. In this way, the percentage of the consistency ratio can be retained to be close to 100%.

4. CONCLUSION

The current paper studies the effects of an incomplete pairwise comparison matrix in an AHP. The study shows that the more consistent a pairwise matrix, the greater the tendency for it to retain its consistency even when one element is missing. This would suggest that the AHP method can be carried out without filling out the entire pairwise comparison matrix, but with one or more missing values.

One method that warrants consideration in future research is the approach used to approximate the missing value. In this study, the calculation of Eigenvectors for the pairwise comparison matrix is considered to be missing. It is expected that an approximation of the missing value can enhance the value of the retained percentage of the consistency ratio.

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