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Fuel Costs Optimization for Long-Haul Flight with Refueling Layovers

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Abstract. This study introduced a mathematical model aimed at optimizing fuel costs for long-haul flights, particularly those requiring refueling. The primary objective was to minimize fuel expenses by considering key factors such as flight routes, aircraft types, refueling points, and refueling quantities. The proposed solution used a 0-1 mixed-integer linear programming (MILP) model, supported by auxiliary variables, to effectively manage the constraints of this optimization problem. The MILP model also considered differences in fuel costs at refueling points, including the departure airport. For validation, a case study was conducted involving a long-haul flight from airport AAA to DDD, with refueling options at airports BBB and CCC. The model effectively determined the most economical flight route, assessed the necessity of refueling, and calculated the required fuel amounts at each refueling location. In summary, this study demonstrated that the proposed model could successfully address the challenges of optimizing fuel costs in long-haul flight.

Keywords: 0-1 mixed integer programming; Long-haul flight; Optimization; Refueling

1. Introduction

Air transport plays a big role in Thailand's economy, acting as a key transportation method for goods and passengers (Office of Industrial Economics, 2021). Its efficiency and ease of use attract many travelers (Wensveen, 2015), creating significant economic benefits for aviation and related industries, such as logistics, tourism, and trade.

Managing operational costs for a long-haul flight that requires refueling presents a critical challenge due to the complexities involved in fuel management and strategic refueling decisions (Hahn, 2012). Fuel costs account for a large proportion of airline operational expenses, directly influencing financial performance (Park and O'Kelly, 2018). Consequently, airlines must optimize fuel consumption and select cost-effective refueling points to balance costs minimization with maintaining an adequate fuel supply. The volatile nature of fuel costs further complicates this process, suggesting the need for an effective model to mitigate financial risks caused by price fluctuations (Hsu and Eie, 2013).

Hedging has proven to be an effective tool for mitigating the volatility of fuel costs. By using financial instruments such as futures, options, and swaps, airlines can stabilize costs and revenue, even during periods of high prices (Swidan, Merkert, and Kwon, 2019; Cobbs and Wolf, 2004). This risk management tool helps airlines safeguard against market unpredictability. Previous investigations also showed a direct correlation between rising fuel costs and increased airfare, which negatively affected passenger demand and

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profitability (Gayle and Lin, 2020). Additionally, delays in ticket payments and uncertainties at key airline hubs make planning flight more complicated, requiring attention to fuel needs, payload limits, and safety considerations (Singh and Sharma, 2015; Wongwiwat, 2014).

Fuel costs are critical in fleet management and aircraft design. Manufacturers focus on improving performance to cut down fuel use and lower operating costs (Elzayady and Elghandour, 2021; Oguntona *et al.,* 2016), enabling airlines to stay competitive (Qiu, Wang, and Qian, 2023). Fleet planning involves factors like fuel price changes, air traffic fees, and infrastructure expenses. For example, turboprop planes cost less per passenger than regional jets (Smirti and Hansen, 2009), making them an alternative for airlines. Also, practices like fuel tankering and ferrying allow airlines to take advantage of price differences between refueling stations (Hubert *et al.,* 2015; Kheraie and Mahmassani, 2012). Improvements in noise prediction have also shaped aircraft designs by addressing noise and environmental regulations (Kusumalestari *et al.,* 2024).

An optimization model has been created to solve fuel consumption and flight route problems. In the context of aircraft routing with air refueling, both heuristic and exact algorithms have been designed to lower costs while ensuring enough fuel (Kannon *et al.,* 2014). One innovative approach, formation flight, has been shown to reduce fuel consumption by 7.7% and operating costs by 2.6% for long international flight (Xu *et al.,* 2014). A graph-based model is another option for optimizing routes when winds change, but it uses a lot of computer power (Bijlsma, 2009). Another method, simulated annealing for mixed airspace, helps make routes shorter and lowers fuel use (Aydoğan and Cetek, 2022). Also, a bi-level optimization framework in formation flight worked well for cutting drag and saving fuel (Xu *et al.,* 2012).

The focus on fuel efficiency and the environment makes the models more necessary. They are especially helpful for cutting CO2 emissions while keeping operations working smoothly (Doulgeris, Kirner, and Laskaridis , 2011). This focus highlights the importance of advanced optimization model in airline operations, which also address environmental issues such as air pollution and waste management (Kondili, 2005). In transportation, such as locomotive assignment, optimization is used to allocate resources efficiently under operational constraints (Piu and Speranza, 2014). Similar models are also applied in fields like telecommunications, computer science, and bioinformatics for solving complex issues (Alba *et al.,* 2009).

Recent advancements in optimization have shown its versatility in resource management across multiple industries. For example, mathematical optimization model such as linear programming (LP) have proven effective in solving production-related challenges (Matousek and Gärtner, 2007). Applications include parking management (Nahry, Tjahjono, and Brotoadi, 2015) and manufacturing, where space allocation helps reduce congestion and production costs (Rosyidi, Fatmawati, and Jauhari, 2016). In environmental efforts, mixed-integer nonlinear programming supports bittern recovery (Widodo *et al.,* 2023), while robust optimization aids in managing plastic waste in Jakarta (Ardi *et al.,* 2023). For airlines, optimization helps with gate assignments (Zhang and Klabjan, 2017), resolves air traffic conflicts (Alonso-Ayuso, *et al.,* 2013), and minimizes delays for better profits (McCarty and Cohn, 2018).

Based on the above description, this study proposes the use of a 0-1 mixed-integer linear programming (MILP) model (Hillier and Lieberman, 2015) to minimize fuel costs for a long-haul flight that requires refueling. The model evaluates various flight scenarios, including direct routes and those involving intermediate stops, while accounting for variations in fuel costs across refueling locations. Additionally, auxiliary variables will be incorporated to ensure compliance with operational constraints. By integrating such elements, this study aims to address critical challenges in fuel costs management and routing efficiency, offering practical solutions for optimizing airline operations.

2. Methods

An LP framework had an objective function and constraints, which were based on a finite number of variables. A 0-1 MILP, which was like LP but more specific (Sioshansi and Conejo, 2017), had variables that were either 0 or 1. These binary variables were important for deciding flight routes, types of aircraft, and where to refuel. Additionally, auxiliary variables were used to enforce the constraints. The 0-1 MILP began by identifying important factors that affect fuel costs, and then provided a detailed explanation of the proposed mathematical framework.

2.1. Identifying Relevant Factors

The calculation of fuel costs was based on various factors (Doganis, 2019), such as flight routes, aircraft types, fuel prices, refueling locations, and fuel consumption rates (Calvet, 2024). To show the proposed model, a small-scale case study was introduced. This study assumed a flight departing from city AAA and heading to city DDD, with three possible routes. The routes were grouped into one direct and two indirect routes that included a refueling layover in city BBB or CCC. Table 1 provided the distances for these three routes, measured in miles.

Table 1 The distance from point to point in miles

Route	Pt. 1 to Pt. 2	Pt. 2 to Pt. 3
R_1: AAA – DDD	5,598	-
R_2: AAA – BBB – DDD	3,050	3,012
R_3: AAA – CCC – DDD	3,100	2,150

This study assumed that there were only three models of aircraft available, including C_1: Boeing 747-400 (B747), C_2: Boeing 777-300ER (B777), and C_3: Airbus A330-300 (A330). Each aircraft had distinct fuel consumption rates and performance metrics. Table 2 detailed relevant data for these aircraft, including their maximum takeoff and landing weights (in tons), maximum fuel capacities (in gallons), ranges with maximum payloads (in nautical miles), and normal cruise speeds (in miles per hour).

Table 2 Aircraft models and their specifications

Aircraft model	B747	B777	A330
Maximum takeoff weight (tons)	394.625	351.534	230
Maximum landing weight (tons)	285.763	251.29	185
Maximum fuel capacity (gallons)	45,714.98	38,428.16	20,608.59
Range with maximum payload (nm.)	5,500	5,700	4,750
Normal cruise speed (miles/hr)	594	576	577.8

Table 3 presented fuel consumption (Hassan, Sobaih, and Salem, 2021) for each aircraft model across the three flight routes, indicating the minimum fuel consumption for the model on each segment of these routes.

Given the analysis of three flight routes and variations in fuel costs across fueling stations in the departure city (AAA) and the two layover cities (BBB and CCC), informed decisions about flight routes, aircraft types, and refueling locations were crucial. Determining the appropriate amount of fuel for each refueling stop while considering these factors was also essential. A 0-1 MILP model addressing this problem was introduced in the following section.

Table 3 Minimum fuel consumption (in gallons) of each model for each route segment.

Route Segment	Aircraft Model	Min. Fuel Consumption
AAA – DDD	B747	35,854.40
	B777	28,890.58
	A330	26,612.44
AAA – BBB	B747	24,149.52
	B777	19,203.98
	A330	14,299.58
BBB – DDD	B747	23,990.66
	B777	19,078.50
	A330	14,095.98
AAA – CCC	B747	15,255.85
	B777	12,190.84
	A330	9,023.62
CCC – DDD	B747	28,510.53
	B777	22,675.78
	A330	18,921.97

2.2. The Proposed Mathematical Model

The mathematical model developed for AAA-DDD flight route case study was designed to optimize flight routes, aircraft types, and refueling locations, as well as fuel quantities at each stop, with the aim of minimizing total fuel costs. The model adopted a systematic design covering three key steps, including defining decision variables, formulating the objective function, and establishing constraints. The constraints ensured that the optimal values of the decision variables satisfied all operational and logistical requirements. The process began by identifying decision and auxiliary variables, along with essential input parameters. These components were then integrated to construct the objective function and specify the constraints.

2.2.1. A Set of Notations:

The notations and their representations in the context of the problem were categorized into decision variables, auxiliary variables, and input parameters as follows.

Decision Variables:

Let R_1, R_2, and R_3 denote the flight routes 1,2, and 3, where:

route R_1 represented the direct route AAA–DDD,

route R_2 was the indirect route AAA–BBB–DDD that refueled at BBB,

route R_3 represented the indirect route AAA–CCC–DDD, refueling at CCC.

$R_i = 1$ when route i was selected and $R_i = 0$ when route i was not selected, where $i = 1, 2, 3$.

Let M_1, M_2 and M_3 denoted the aircraft models B747, B777, and A330, respectively.

$M_j = 1$ when aircraft model j was selected and $M_j = 0$ when model j was not selected, where $j = 1, 2, 3$

Let P_1, P_2, and P_3 represented refueling cities AAA, BBB, and CCC, respectively.

$P_k = 1$ when refueling city k was selected and $P_k = 0$ if city k was not selected, where $k = 1, 2, 3$.

$F_{ijk} \geq 0$ represented the quantity of fuel, in gallons, refueled on the selected route i using aircraft model j at refueling city k, where $i,j,k = 1, 2, 3$.

Auxiliary Variables:

H_{ijk} = 1 when route *i* with aircraft model *j*, and refueling city *k* was selected.

H_{ijk} = 0 when route *i* with aircraft model *j* and refueling city *k* was not selected,

where i,j,k = 1, 2, 3.

G_{ij} = 1 when route *i* with aircraft model *j* was selected.

G_{ij} = 0 when route *i* with aircraft model *j was* not selected, where i,j = 1, 2, 3.

Input Parameters

Let α, β, and γ denoted fuel costs at refueling cities AAA, BBB, and CCC, respectively.

The auxiliary variables H_{ijk} and G_{ij} were introduced to ensure that fuel consumption rates and aircraft tank capacities were in line with the relevant aircraft specifications.

<u>2.2.2. The Objective Function:</u>

The objective of this model was to minimize the total fuel expenses as shown in the following Equation (1), i.e.,

$$\text{Minimize } \alpha F_{111} + \alpha F_{121} + \alpha F_{131} + \alpha F_{211} + \beta F_{212} + \alpha F_{221} + \beta F_{222} + \alpha F_{231} + \beta F_{232} + \alpha F_{311} + \gamma F_{313} + \alpha F_{321} + \gamma F_{323} + \alpha F_{331} + \gamma F_{333} \quad (1)$$

<u>2.2.3. The Constraints:</u>

Equation (2) specified that while three flight routes were available for selection, only one could be selected.

$$R_1 + R_2 + R_3 = 1 \quad (2)$$

Equation (3) indicated that, although three aircraft models were available for selection, only one was chosen.

$$M_1 + M_2 + M_3 = 1 \quad (3)$$

Equation (4) through (8) defined the constraints related to refueling points, as explained below.

Since AAA was the departure city, all aircraft needed to be fueled at P_1.

$$P_1 = 1 \quad (4)$$

For flight routes with layovers, refueling at the layover cities P_2 and P_3 was optional.

$$P_1 + P_2 + P_3 \leq 2 \quad (5)$$

Route R_1 was a direct flight route, making refueling only occur at the departure city.

$$R_1 + P_2 + P_3 \leq 1 \quad (6)$$

Routes R_2 and R_2 involved layover cities, making it possible to be refueled or not, as specified in equations (7) and (8).

$$P_2 \leq R_2 \quad (7)$$

$$P_3 \leq R_3 \quad (8)$$

For each scenario, which included a selected flight route, aircraft types, and refueling location, the required fuel quantity was determined using auxiliary 0-1 variables (H_{ijk}). The variables ensured that fuel quantity satisfied the minimum fuel consumption required for aircraft types on each segment of the selected route while staying in the maximum fuel tank capacity, as defined in Equations (9) through (38).

$$R_1 + M_1 + P_1 - 2 \leq 3H_{111} \leq R_1 + M_1 + P_1 \quad (9)$$

$$35{,}854.40H_{111} \le F_{111} \le 45{,}714.98H_{111} \quad (10)$$
$$R_1 + M_2 + P_1 - 2 \le 3H_{121} \le R_1 + M_2 + P_1 \quad (11)$$
$$28{,}890.58H_{121} \le F_{121} \le 38{,}428.16H_{121} \quad (12)$$
$$R_1 + M_3 + P_1 - 2 \le 3H_{131} \le R_1 + M_3 + P_1 \quad (13)$$
$$26{,}612.44H_{131} \le F_{131} \le 20{,}608.59H_{131} \quad (14)$$
$$R_2 + M_1 + P_1 - 2 \le 3H_{211} \le R_2 + M_1 + P_1 \quad (15)$$
$$24{,}149.52H_{211} \le F_{211} \le 45{,}714.98H_{211} \quad (16)$$
$$R_2 + M_1 + P_2 - 2 \le 3H_{212} \le R_2 + M_1 + P_2 \quad (17)$$
$$H_{212} \le F_{212} \le 45{,}714.98H_{212} \quad (18)$$
$$R_2 + M_2 + P_1 - 2 \le 3H_{221} \le R_2 + M_2 + P_1 \quad (19)$$
$$19{,}203.98H_{221} \le F_{221} \le 38{,}428.16H_{221} \quad (20)$$
$$R_2 + M_2 + P_2 - 2 \le 3H_{222} \le R_2 + M_2 + P_2 \quad (21)$$
$$H_{222} \le F_{222} \le 38{,}428.16H_{222} \quad (22)$$
$$R_2 + M_3 + P_1 - 2 \le 3H_{231} \le R_2 + M_3 + P_1 \quad (23)$$
$$14{,}229.58H_{231} \le F_{231} \le 20{,}608.59H_{231} \quad (24)$$
$$R_2 + M_3 + P_2 - 2 \le 3H_{232} \le R_2 + M_3 + P_2 \quad (25)$$
$$H_{232} \le F_{232} \le 20{,}608.59H_{232} \quad (26)$$
$$R_3 + M_1 + P_1 - 2 \le 3H_{311} \le R_3 + M_1 + P_1 \quad (27)$$
$$15{,}255.85H_{311} \le F_{311} \le 45{,}714.98H_{311} \quad (28)$$
$$R_3 + M_1 + P_3 - 2 \le 3H_{313} \le R_3 + M_1 + P_3 \quad (29)$$
$$H_{313} \le F_{313} \le 45{,}714.98H_{313} \quad (30)$$
$$R_3 + M_2 + P_1 - 2 \le 3H_{321} \le R_3 + M_2 + P_1 \quad (31)$$
$$12{,}190.84H_{321} \le F_{321} \le 38{,}428.16H_{321} \quad (32)$$
$$R_3 + M_2 + P_3 - 2 \le 3H_{323} \le R_3 + M_2 + P_3 \quad (33)$$
$$H_{323} \le F_{323} \le 38{,}428.16H_{323} \quad (34)$$
$$R_3 + M_3 + P_1 - 2 \le 3H_{331} \le R_3 + M_3 + P_1 \quad (35)$$
$$9{,}023.62H_{331} \le F_{331} \le 20{,}608.59H_{331} \quad (36)$$
$$R_3 + M_3 + P_3 - 2 \le 3H_{333} \le R_3 + M_3 + P_3 \quad (37)$$
$$H_{333} \le F_{333} \le 20{,}608.59H_{333} \quad (38)$$

When selecting flight routes R_2 and R_3, refueling needed to occur both at the departure city and the layover city. The combined fuel quantity, controlled by the auxiliary 0-1 variables (G_{ij}), must equal or exceed the total fuel consumption required for the entire route, as described in Equations (39) through (50).

$$R_2 + M_1 - 1 \le 2G_{21} \le R_2 + M_1 \quad (39)$$
$$48{,}140.18G_{21} \le F_{211} + F_{212} \quad (40)$$
$$R_2 + M_2 - 1 \le 2G_{22} \le R_2 + M_2 \quad (41)$$
$$38{,}282.48\, G_{22} \le F_{221} + F_{222} \quad (42)$$
$$R_2 + M_3 - 1 \le 2G_{23} \le R_2 + M_3 \quad (43)$$
$$28{,}325.56G_{23} \le F_{231} + F_{232} \quad (44)$$
$$R_3 + M_1 - 1 \le 2G_{31} \le R_3 + M_1 \quad (45)$$
$$43{,}766.38G_{31} \le F_{311} + F_{313} \quad (46)$$
$$R_3 + M_2 - 1 \le 2G_{32} \le R_3 + M_2 \quad (47)$$
$$34{,}866.62G_{32} \le F_{321} + F_{323} \quad (48)$$
$$R_3 + M_3 - 1 \le 2G_{33} \le R_3 + M_3 \quad (49)$$
$$27{,}945.59G_{33} \le F_{331} + F_{333} \quad (50)$$

For indirect flight routes R_2 and R_3, the total fuel quantity combined from the departure and layover cities must not surpass fuel tank capacity of the aircraft types being used, as detailed in Equations (51) through (56).

$$F_{211} - 24{,}149.52 + F_{212} \leq 45{,}714.98 \quad (51)$$
$$F_{221} - 19{,}203.98 + F_{222} \leq 38{,}428.16 \quad (52)$$
$$F_{231} - 14{,}229.58 + F_{232} \leq 20{,}608.59 \quad (53)$$
$$F_{311} - 15{,}255.85 + F_{313} \leq 45{,}714.98 \quad (54)$$
$$F_{321} - 12{,}190.84 + F_{323} \leq 38{,}428.16 \quad (55)$$
$$F_{331} - 9{,}023.62 + F_{333} \leq 20{,}608.59 \quad (56)$$

The decision variable (F_{ijk}), representing fuel quantity, was required to always be greater than or equal to 0. Other decision variables, which indicated whether a particular option was selected, must take on values of either 0 or 1, as outlined in Equations (57) and (58).

$$F_{ijk} \geq 0 \quad (57)$$
$$R_i, M_j, P_k, H_{ijk}, G_{ij} = 0 \text{ or } 1 \quad (58)$$

3. Results and Discussion

Assuming fuel prices at refueling points AAA, BBB, and CCC were set at $1.24, $1.15, and $1.33 per gallon, respectively, a detailed analysis was performed to evaluate eight potential fuel cost scenarios based on the model in the case study. The scenarios accounted for various combinations of flight routes, aircraft types, and refueling locations. The results, which showed fuel quantities to be refueled at the starting point of each route segment for all flight routes and aircraft types, along with their corresponding total fuel costs, were presented in Table 4.

In Scenario 3, where the flight followed an indirect route from AAA to DDD with a layover in BBB, using the B747 aircraft, refueling quantities at AAA and BBB were 24,149.52 gallons and 3,990.66 gallons, respectively. The lowest total fuel cost for this scenario was $57,534.66. Despite Scenarios 6 and 7 being indirect routes with layovers in CCC, the aircraft did not require refueling at CCC, as fuel loaded at AAA was sufficient to complete the entire route.

Table 4 The refueled quantities at the starting point of each route segment for both direct and indirect flights, across various aircraft types, along with their corresponding total fuel costs

Scenario#	Route	Route Seg.	Aircraft	Refueled at Start (Gal.)	$Fuel Cost
1	Direct	AAA – DDD	B747	35,584.40	$44,459.46
2	Direct	AAA – DDD	B777	28,890.58	$35,824.32
3	Indirect 1	AAA – BBB BBB – DDD	B747	24,149.52 23,990.66	$57,534.66
4	Indirect 1	AAA – BBB BBB – DDD	B777	19,203.98 19,078.50	$45,753.21
5	Indirect 1	AAA – BBB BBB – DDD	A330	14,229.58 14,095.98	$33,855.06
6	Indirect 2	AAA – CCC CCC – DDD	B747	43,766.38 0.00	$54,270.31
7	Indirect 2	AAA – CCC CCC – DDD	B777	34,866.62 0.00	$43,234.61
8	Indirect 2	AAA – CCC CCC – DDD	A330 A330	20,608.59 7,337.00	$35,312.86

The optimal solutions obtained from the proposed model were detailed in Table 5, with the final values of all decision variables. The results showed the decision variables R_2, M_3, P_1, and P_2 were assigned a value of 1, indicating that the most cost-effective configuration involved selecting flight route R_2: AAA-BBB-DDD, using aircraft types M_3: A330-300.

According to the optimal plan, the aircraft was first refueled at the departure city (P_1: AAA), with a total of F_{231} = 14,229.58 gallons. It was then refueled at the layover city (P_2: BBB), with a total of F_{232} = 14,095.98 gallons. The configuration results in the lowest possible fuel cost of $33,855.06 was calculated as (14,229.58 gallons x $1.24 per gallon) + (14,095.98 gallons x $1.15 per gallon). The results showed the optimization model's effectiveness in minimizing fuel costs while considering various factors.

When fuel prices at refueling locations differ from the fixed values assumed in this study, the optimal solution might change accordingly. This could affect the selection of flight routes, aircraft types, refueling locations, and fuel quantities. However, the proposed model remained adaptable and robust, capable of handling varying price conditions and scaling up for larger or more complex problems without compromising effectiveness.

Table 5 Final values of the decision variables

Variable	Value	Variable	Value (Gal.)	Variable	Value (Gal.)
R_1	0	F_{111}	0	F_{311}	0
R_2	1	F_{121}	0	F_{321}	0
R_3	0	F_{131}	0	F_{323}	0
M_1	0	F_{211}	0	F_{331}	0
M_2	0	F_{212}	0	F_{333}	0
$\boldsymbol{M_3}$	1	F_{221}	0		
$\boldsymbol{P_1}$	1	F_{222}	0		
$\boldsymbol{P_2}$	1	F_{231}	14,229.58		
$\boldsymbol{P_3}$	0	F_{232}	14,095.98		

4. Conclusions

In conclusion, this study talked about a 0-1 MILP model that was used to lower the fuel costs for flight with passengers. The model looked at important factors, like which flight routes to take, what kinds of planes to use, fuel prices at refueling spots, and how much fuel was used. The 0-1 MILP had a setup that helped to compare different options, making the cheapest way to be picked. The model chose the best flight route, checked if refueling was needed, and decided where it would be best to refuel. Also, the 0-1 MILP figured out exactly how much fuel would be needed at every stop, making sure the plane could finish its trip without carrying too much fuel or spending extra money. This model helped save money and made operations better. One limitation of the study was the assumption of fixed fuel costs at refueling locations. However, the act of being flexible allowed the designed model to be updated with new price inputs, maintaining its relevance even under fluctuating market conditions. A 0-1 MILP could also be scaled and adapted to address real-world and large-scale problems by adjusting specific constraints. The flexibility made it a valuable tool for flight planning, enabling detailed analysis and comparison of fuel costs across different flight routes, aircraft types, and refueling points, ultimately contributing to lower operational costs. The proposed model offered significant environmental benefits. By optimizing fuel consumption and refueling strategies, it helped reduce fuel burn and lower carbon emissions. Identifying more fuel-efficient routes and minimizing unnecessary refueling contributed to decreasing the environmental impact of air travel. Furthermore, the model could be adjusted to prioritize greener aircraft models or alternative fuels, promoting sustainability in aviation. This mathematical model was in line with cost efficiency with environmental objectives, reducing both operational costs and emissions.

Conflict of Interest

The authors declare no conflicts of interest.

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