International Journal of Technology 16(1) 160-175 (2025)

International Journal of Technology

http://ijtech.eng.ui.ac.id

Research Article

Mixed Balanced Truncation for Reducing the Complexity of Large-Scale Electrical and Electronic System Simulations

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Abstract: This study focuses on Model Order Reduction (MOR) to optimize such systems' simulation and analysis capabilities in the context of increasingly complex electrical and electronic systems, coupled with computational and processing resource limitations. This paper proposes the Mixed Balanced Truncation (MBT) algorithm, which combines the strengths of Balanced Truncation (BT) and Positive-Real Balanced Truncation (PRBT) while addressing their respective limitations. The MBT algorithm is developed based on Lyapunov and Riccati equations, ensuring the stability and passivity of the reduced-order system. The proposed method is validated through large-scale electrical circuit systems, using RLC network models as illustrative examples. The results demonstrate that MBT achieves effective order reduction with minimal error while reducing computational costs. The main contributions of this work in developing the new reduction algorithm include the introduction of a novel definition of mixed balanced systems and theoretical advancements through the development of theorems, lemmas, and corollaries accompanied by rigorous mathematical proofs. This study makes significant theoretical contributions and provides practical solutions for designing, modeling, and reducing the complexity of electrical and electronic systems, particularly passive linear systems in general.

Keywords: Computational Efficiency; Descriptor Systems; Electrical Circuit Simulation; Mixed Balanced Truncation; Model Order Reduction

1. Introduction

In circuit simulation, Modified Nodal Analysis (MNA) is a widely used method for constructing mathematical models of circuit behavior (Choupanzadeh and Zadehgol, 2023; Pavan and Temes, 2023; Hao and Shi, 2022; Günther et al., 2005). This technique represents the input-output relationship of a circuit as a linear descriptor system, expressed by the following equation (1).

$$\begin{cases} \boldsymbol{E}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \end{cases} \Leftrightarrow \boldsymbol{G}(s) := \boldsymbol{D} + \boldsymbol{C}(s\boldsymbol{E} - \boldsymbol{A})^{-1}\boldsymbol{B} \tag{1}$$

where $\boldsymbol{E} \in \mathbb{R}^{nxn}, \boldsymbol{D} \in \mathbb{R}^{mxm}, \boldsymbol{C} \in \mathbb{R}^{mxn}, \boldsymbol{B} \in \mathbb{R}^{nxm}, \boldsymbol{A} \in \mathbb{R}^{nxn}, \boldsymbol{x}(t) \in \mathbb{R}^{n}, \boldsymbol{u}(t) \in \mathbb{R}^{m}, \boldsymbol{y}(t) \in \mathbb{R}^{m}$

This work was supported by the Program of the Ministry of Education and Training of Vietnam under grant number B2023-TNA-17

https://doi.org/10.14716/ijtech.v16i1.7180
Received July 2024; Revised November 2024; Accepted December 2024; Published January 2025

Remark 1. The system described by equation (1) exhibits several key characteristics and requirements (these properties ensure the system meets the demands for simulation, analysis, and the application of model reduction algorithms):

- The system is minimal, ensuring no redundant states.
- The dynamics are stable and passive, indicating that the system does not generate energy and all eigenvalues of the pencil matrix pair ($\mathbf{E}$, $\mathbf{A}$) have non-positive real parts.
- The initial conditions are such that the state variables, inputs, and outputs are all zero.
- Matrices $\mathbf{A}$ and $\mathbf{E}$ are nonsingular, with ranks equal to n, and matrices $\mathbf{A}$ and $\mathbf{E}^{-1}\mathbf{A}$ are stable.
- The matrix $\mathbf{D}$ satisfies the condition $\mathbf{D} + \mathbf{D}^T \geq 0$, ensuring that the system's output matrix is positive semi-definite.

Dimensionality Reduction (DR) or Model Order Reduction (MOR) is a crucial technique widely applied in mathematical modeling across various fields, including electrical and electronic systems (Benner et al., 2021; Benner et al., 2020; Fortuna et al., 2012; Schilders et al., 2008). The primary goal of DR and MOR is to simplify a complex, high-dimensional model by replacing it with a lower-dimensional model while preserving the system's essential physical properties and dynamic behavior. This simplification significantly reduces computational complexity, enabling faster computations required for real-time simulations. Furthermore, it reduces computational workload and storage requirements, enhancing hardware performance and cost efficiency, especially in resource-constrained environment.

In large-scale circuit simulations, many model reduction algorithms are applied in various critical applications. Among them, methods such as Krylov subspace (Freund, 2022; Freund, 2000), Rational Krylov (Ali et al., 2019), Moment Matching (Benner and Feng, 2021; Prajapati and Prasad, 2020), Asymptotic Waveform Evaluation (Jiang and Yang, 2021), Lanczos technique (Wittig et al., 2002), Arnoldi iteration (Song et al., 2017; Jiang and Xiao, 2015), Proper Orthogonal Decomposition (Gräßle et al., 2020; Manthey et al., 2019), Singular Value Decomposition (Younes et al., 2021), Principal Component Analysis (Anaparthi et al., 2005), Padé approximation (Singh et al., 2008), Singular Perturbation (Khan et al., 2019; Huisinga and Hofmann, 2018), Matrix Interpolation (Kassis et al., 2016; Samuel et al., 2014), frequency weighting balance truncation (Floros et al., 2019; Rydel and Stanisławski, 2018), time weighting balance truncation (König and Freitag, 2023), and others stand prominent (Kumar and Ezhilarasi, 2023a; Gugercin and Antoulas, 2004).

During the exploration of foundational methodologies prior to developing a new model order reduction algorithm, the research team focused particularly on two techniques: Balanced Truncation (BT) (Axelou et al., 2023; Kumar and Ezhilarasi, 2023b; Hossain and Trenn, 2023; Suman and Kumar, 2021; Grussler et al., 2021; Antoulas, 2005; Mehrmann and Stykel, 2005) and Positive-Real Balanced Truncation (PRBT) (Poort et al., 2023; Breiten and Unger, 2022; Zulfiqar et al., 2018; Benner and Stykel, 2017; Reis and Stykel, 2010; Reis and Stykel, 2010; Tan and He, 2007; Tan and He, 2007). These methods were prioritized due to their ability to preserve the physical properties of the original system, specifically stability (for BT) and passivity (for PRBT). Both BT and PRBT ensure that reduced-order models retain these crucial physical characteristics, which are essential for circuit simulations.

Balanced Truncation (BT), a classical approach to MOR, was first introduced by Moore in 1981. This method has been extensively studied, refined, and applied across various applications (Axelou et al., 2023; Kumar and Ezhilarasi, 2023b; Hossain and Trenn, 2023; Suman and Kumar, 2021; Grussler et al., 2021; Antoulas, 2005; Mehrmann and Stykel, 2005). BT operates by balancing the controllability and observability Gramians of the system and then truncating states with small singular values. This technique ensures system stability is preserved and achieves minimal reduction error, particularly in cases of moderate-order reduction. However, a notable limitation of BT is its inability to maintain the passivity of the original system, which is critical in practical applications, such as ensuring that electrical circuits do not generate energy (Breiten and Unger, 2022).

To address the limitations of BT, the Positive-Real Balanced Truncation (PRBT) method was developed for positive-real systems (Reis and Stykel, 2010; Tan and He, 2007). PRBT enables model order reduction while preserving the system's passivity, a property that ensures the system cannot generate or amplify energy. Numerous studies have explored improvements and applications of PRBT, highlighting its significance in various fields (Poort et al., 2023; Breiten and Unger, 2022; Zulfiqar et al., 2018; Benner and Stykel, 2017). Although PRBT preserves essential physical properties such as stability and passivity, it often leads to larger reduction errors compared to BT and involves solving more complex problems, resulting in higher computational costs.

Several hybrid methods have been proposed to combine the strengths of BT and PRBT while mitigating their respective weaknesses (Salehi et al., 2022; Salehi et al., 2021a; 2021b; 2021c; Zulfiqar et al., 2020; Lindmark and Altafini, 2017; Zulfiqar et al., 2017; Unneland et al., 2007a; 2007b; Phillips et al., 2002). These methods often involve complex balancing techniques or employ mixed Gramians. These methods often rely on intricate balancing techniques or the use of mixed Gramians. However, their applicability is typically restricted to standard linear systems, rendering them unsuitable for direct application to linear descriptor systems, which are frequently encountered in circuit analysis models. Furthermore, methods described in studies such as (Salehi, Karimaghaee and Khooban, 2021a; Salehi et al., 2022; Salehi, Karimaghaee, and Khooban, 2021b) require solving two Riccati equations, which adds computational complexity and cost.

To overcome these challenges, we propose a novel algorithm named Mixed Balanced Truncation (MBT), specifically designed to reduce the order of continuous-time descriptor systems in the circuits model. MBT leverages the advantages of both BT and PRBT by incorporating techniques that ensure the system remains stable and passive while minimizing computational costs and reducing errors. This novel approach addresses the limitations of previous methods and provides a more efficient solution for DR in practical applications.

The primary contributions of this paper include a comprehensive definition of the proposed methods, three theorems that establish the theoretical foundation, two lemmas, two corollaries, and a new algorithm. The effectiveness and applicability of the proposed MBT method are demonstrated through illustration examples and simulations. This study advances the current body of literature by providing new insights and practical solutions for model order reduction in large-scale electronic circuit simulations, representing a significant step forward in this research field. This research contributes to the theoretical understanding of MOR. It offers a practical algorithm that can be applied to improve the efficiency and performance of electronic and electrical systems. The proposed MBT method paves the way for more effective and reliable simulations, crucial for designing and analyzing modern complex systems.

2. Preliminaries

2.1. Balanced Truncation (BT) for reducing model order

The Balanced Truncation (BT) algorithm is constructed on the principle of balancing the Grammians of the system. This involves using a non-singular transformation matrix to equalize and diagonalize the controllability and observability of Grammians. The reduced-order model is then obtained by eliminating the modes associated with minor Hankel singular values, which represent low-energy modes with minimal influence on the system's behavior. The implementation details of the BT algorithm are presented in (Axelou et al., 2023; Kumar and Ezhilarasi, 2023b; Hossain and Trenn, 2023; Suman and Kumar, 2021; Grussler et al., 2021; Mehrmann and Stykel, 2005; Antoulas, 2005).

Theorem 1 (Antoulas, 2005). If the system described by Equation (1) is stable, then the matrices $\mathbf{K}_c$ (controllability Gramian) and $\mathbf{K}_o$ (observability Gramian) are symmetric and positive definite. These matrices satisfy the following Lyapunov equations (2) and (3).

$$\boldsymbol{A K_c E^T + E K_c A^T = -B B^T} \tag{2}$$

$$\boldsymbol{A^T K_o E + E^T K_o A = -C^T C} \tag{3}$$

2.2. Positive-Real Balanced Truncation (PRBT) for reducing model order

The Positive-Real Balanced Truncation (PRBT) algorithm extends the principles of BT to address passive systems specifically. In this technique, the matrices $\mathbf{J}_c$ (control Gramian) and $\mathbf{J}_o$ (observation Gramian) are computed by solving two Riccati equations. The implementation details of the PRBT algorithm are presented in (Poort et al., 2023; Breiten and Unger, 2022; Zulfiqar et al., 2018; Benner and Stykel, 2017; Reis and Stykel, 2010; Reis and Stykel, 2010; Tan and He, 2007; Tan and He, 2007).

Theorem 2 (Reis and Stykel, 2010; Tan and He, 2007). A system described by equation (1) exhibits passivity if and only if its transfer function G(s) is of the positive-real type. This condition is met if there exist matrices Jc and Jo that satisfy the following Riccati equations (4) and (5).

$$\boldsymbol{AJ_cE^T + EJ_cA^T = (EJ_cC^T - B)inv(D + D^T)(B - EJ_cC^T)^T} \quad (4)$$

$$\boldsymbol{A^TJ_oE + E^TJ_oA = (E^TJ_oB - C^T)inv(D + D^T)(C^T - E^TJ_oB)^T} \quad (5)$$

These equations ensure that the system maintains passivity while reducing its order, effectively preserving the original system's essential characteristics.

3. Reduction of Model Order Using Mixed Balanced Truncation

The Mixed Balanced Truncation algorithm is utilized to reduce the model order of mixed-balanced systems. Therefore, it is first necessary to determine whether the system in question conforms to this balanced form. A system is said to satisfy the mixed balanced property if it fulfills the criteria specified in Definition 1.

Definition 1. A linear descriptor system described by equation (1) satisfying Remark 1 is termed a mixed-balanced system if it meets the following conditions (6) or (7).

$$\boldsymbol{K_c = K_c^T = J_o = J_c^T = X_B = diag\,(\psi_1, \psi_2, \ldots, \psi_n)} \quad (6)$$

$$\boldsymbol{J_o = J_o^T = K_c = K_c^T = X_B = diag\,(\psi_1, \psi_2, \ldots, \psi_n)} \quad (7)$$

where $\mathbf{K}_c$ and $\mathbf{J}_o$ satisfy Equations (2) and (5), and $\mathbf{J}_c$ and $\mathbf{K}_o$ satisfy Equations (3) and (4). $(\psi_1, \psi_2, \ldots, \psi_n)$are the Hankel singular values of the mixed-balanced system with $\psi_1, \psi_2, \ldots, \psi_n > 0$.

Remark 2. If the system described by equation (1) does not satisfy the criteria defined in Definition 1, then it is possible to convert this system into a mixed-balanced system using Theorem 3.

Theorem 3. Consider the system described by equation (1) satisfying the conditions outlined in Remark 1. A non-singular transformation always exists via a transformation matrix $\mathbf{T}_z$, such as equation (8) or (9).

$$\boldsymbol{T_z^TJ_oT_z = T_z^{-1}K_cT_z^{-T} = X_B = diag\,(\psi_1, \psi_2, \ldots, \psi_n)} \quad (8)$$

$$\boldsymbol{T_z^TK_oT_z = T_z^{-1}J_cT_z^{-T} = X_B = diag\,(\psi_1, \psi_2, \ldots, \psi_n)} \quad (9)$$

Proof of Theorem 3. By applying the Cholesky decomposition to $\mathbf{K}_c$ and $\mathbf{J}_o$, followed by performing Singular Value Decomposition on the matrix $\boldsymbol{P^TQ}$, being the Cholesky factors, we then calculate $\mathbf{T}_z$ and its inverse. This results in equations (10) and (11).

$$\begin{aligned}\boldsymbol{T_z^{-1}K_cT_z^{-T}} &= \boldsymbol{X_B^{-1/2}S_1^TQ^T \times PP^T \times QS_1X_B^{-1/2}}\\ &= \boldsymbol{X_B^{-1/2}S_1^T \times (Q^TP) \times (P^TQ) \times S_1X_B^{-1/2}}\\ \Leftrightarrow \boldsymbol{T_z^{-1}K_cT_z^{-T}} &= \boldsymbol{X_B^{-1/2}S_1^T \times S_1X_BS_2^T \times S_2X_BS_1^T \times S_1X_B^{-1/2}}\\ \Leftrightarrow \boldsymbol{T_z^{-1}K_cT_z^{-T}} &= \boldsymbol{X_B^{-1/2} \times X_B \times X_B \times X_B^{-1/2} = X_B = diag\,(\psi_1, \psi_2, \ldots, \psi_n)}\end{aligned} \quad (10)$$

$$\begin{aligned}&\boldsymbol{T_z^TJ_oT_z = X_B^{-1/2}S_2^TP^T \times QQ^T \times PS_2X_B^{-1/2} = X_B^{-1/2}S_2^T \times (P^TQ) \times (Q^TP) \times S_2X_B^{-1/2}}\\ &\Leftrightarrow \boldsymbol{T_z^TJ_oT_z = X_B^{-1/2}S_2^T \times S_2X_BS_1^T \times S_1X_BS_2^T \times S_2X_B^{-1/2}}\\ &\Leftrightarrow \boldsymbol{T_z^TJ_oT_z = X_B^{-1/2} \times X_B \times X_B \times X_B^{-1/2} = X_B = diag\,(\psi_1, \psi_2, \ldots, \psi_n)}\end{aligned} \quad (11)$$

Lemma 1. For a system described by equation (1) satisfying Remark 1, the eigenvalues of the matrices $\mathbf{K}_c\mathbf{J}_o$ or $\mathbf{J}_c\mathbf{K}_o$ are positive and remain invariant under non-singular transformations facilitated by the matrix $\mathbf{T}_z$.

Proof of Lemma 1. By performing the diagonalization of the matrix product K_cJ_o, we derive $\boldsymbol{K_cJ_o} = \boldsymbol{T_z \Lambda T^{-1}}_z$, where $\boldsymbol{\Lambda}$ is a diagonal matrix containing the eigenvalues ρ_i of K_cJ_o; T_z represents a matrix with eigenvectors of K_cJ_o as its columns. Additionally, we have equation (12).

$$\begin{aligned}&\boldsymbol{K_cJ_o} = \boldsymbol{PP^T} \times \boldsymbol{QQ^T} = \boldsymbol{P} \times (\boldsymbol{Q^TP})^{\boldsymbol{T}} \times (\boldsymbol{Q^TP}) \times \boldsymbol{P^{-1}}\\ &\Leftrightarrow \boldsymbol{K_cJ_o} = (\boldsymbol{T_zM_B}^{1/2}\boldsymbol{S_2}^{-1}) \times (\boldsymbol{S_2X_BS_1}^{\boldsymbol{T}}) \times (\boldsymbol{S_1X_BS_2}^{\boldsymbol{T}}) \times (\boldsymbol{S_2X_B}^{-1/2}\boldsymbol{T_z}^{-1}) \Leftrightarrow \boldsymbol{K_cJ_o}\\ &\quad = \boldsymbol{T_zX_B}^2\boldsymbol{T_z}^{-1}\end{aligned} \tag{12}$$

Thus, we can infer that $\rho_i = \psi_i^2$, where ψ_i are the Hankel singular values of the mixed-balanced system, satisfying $\psi_i^2 > 0$. Therefore, Lemma 1 is proven.

Lemma 2. For a system described by equation (1) that meets the conditions in Remark 1, achieving a mixed-balanced state through a non-singular transformation using the matrix T_z is always possible, resulting in the new system matrices described by equation (13).

$$\boldsymbol{E_X} = \boldsymbol{T_z}^{-1}\boldsymbol{ET_z}; \boldsymbol{A_X} = \boldsymbol{T_z}^{-1}\boldsymbol{AT_z}; \boldsymbol{B_X} = \boldsymbol{T_z}^{-1}\boldsymbol{B}; \boldsymbol{C_X} = \boldsymbol{CT_z}; \boldsymbol{D_X} = \boldsymbol{D} \tag{13}$$

Proof of Lemma 2. When equations (8) and (13) are substituted into equations (2) and (5), the updated system is described by equations (14) and (15).

$$\boldsymbol{A_XX_BE_X}^{\boldsymbol{T}} + \boldsymbol{E_XX_BA_X}^{\boldsymbol{T}} = -\boldsymbol{B_XB_X}^{\boldsymbol{T}} \tag{14}$$

$$\boldsymbol{A_X}^{\boldsymbol{T}}\boldsymbol{X_BE_X}, +\boldsymbol{E_X}^{\boldsymbol{T}}\boldsymbol{X_BA_X} = (\boldsymbol{E_X}^{\boldsymbol{T}}\boldsymbol{X_BB_X} - \boldsymbol{C_X}^{\boldsymbol{T}})(\boldsymbol{D_X} + \boldsymbol{D_X}^{\boldsymbol{T}})^{-1}(\boldsymbol{C_X}^{\boldsymbol{T}} - \boldsymbol{E_X}^{\boldsymbol{T}}\boldsymbol{X_BB_X})^{\boldsymbol{T}} \tag{15}$$

These new equations (14) and (15) possess solutions that meet the criteria of Definition 1. Therefore, by employing the equivalent transformation with the non-singular matrix T_z, the original system converts into the mixed-balanced system, thereby establishing the validity of Lemma 2.

Algorithm 1. Reduce Model Order Using Mixed Balanced Truncation

Input: The dynamical system G(s) described by equation 1 satisfying the conditions of Remark 1.
Output: Reduced-order system described by the reduced matrices: $(\mathbf{E}_r, \mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r, \mathbf{D}_r)$

Equivalent transformation of the original system to a mixed-balanced system:

1. Compute K_c and J_o from equations (2) and (5).
2. Perform Cholesky decomposition on K_c and J_o as equations (16) and (17).

$$\boldsymbol{K_c} = \boldsymbol{PP}^T \tag{16}$$

$$\boldsymbol{J_o} = \boldsymbol{QQ}^T \tag{17}$$

where **P** and **Q** are invertible lower triangular matrices.

3. Decompose the singular values of the product $\boldsymbol{P^TQ}$ as equation (18).

$$\boldsymbol{P^TQ} = \boldsymbol{S}_1\boldsymbol{X}_B\boldsymbol{S}_2{}^T \tag{18}$$

where **U** and **V** are orthogonal matrices, and X_B is a diagonal matrix containing singular values.

4. Calculate the conversion matrix T_z and inverse according to equations (19) and (20).

$$\boldsymbol{T}_z = \boldsymbol{PS}_2\boldsymbol{X}_B{}^{-1/2} \tag{19}$$

$$\boldsymbol{T}_z{}^{-1} = \boldsymbol{X}_B{}^{-1/2}\boldsymbol{S}_1{}^T\boldsymbol{Q}^T \tag{20}$$

5. Calculate the updated system matrices of the mixed-balanced system using the conversion equation (13).

Reduce Model Order Using Mixed Balanced Truncation:

6. Choose the intended reduced dimension r where 0 < r < n.
7. Compute the projection matrices as equations (26) and (27).

$$\boldsymbol{V}_{MBT} = \mathbf{T}_z(:,1:\mathrm{r}) \tag{21}$$

$$\boldsymbol{W}_{MBT} = \mathbf{T}_z{}^{-1}(1:\mathrm{r},:)^T; \tag{22}$$

8. Calculate the matrices of the reduced order system as in the equations in (23).

$$\boldsymbol{E}_r = \boldsymbol{W}_{MBT}{}^T\boldsymbol{EV}_{MBT}; \boldsymbol{A}_r = \boldsymbol{W}_{MBT}{}^T\boldsymbol{AV}_{MBT}; \boldsymbol{B}_r = \boldsymbol{W}_{MBT}{}^T\boldsymbol{B}; \boldsymbol{C}_r = \boldsymbol{CV}_{MBT}; \boldsymbol{D}_r = \boldsymbol{D} \tag{23}$$

Remark 3. In this algorithm, we utilize the Gramians K_c and J_o. Alternatively, we can use the Gramians J_c and K_o owing to the balanced and symmetric nature of the system.

Corollary 1. The mixed-balanced system obtained from Algorithm 1 (from Step 1 to Step 5) retains the properties described in Remark 1. The control matrix X_{Bc} and the observer matrix X_{Bo} of the mixed-balanced system are symmetric, positive definite diagonal matrices, as in expression (24).

$$X_{Bc} = X_{Bo} = X_B = diag\ (\psi_1, \psi_2, \ldots, \psi_n) \tag{24}$$

Proof of corollary 1. To verify the stability and passivity of the resulting mixed-balanced system, we solve the Lyapunov equation for the new control matrix X_{Bc} and the Riccati equation for the new observation matrix X_{Bo}, as specified in equations (25) and (26), respectively.

$$\boldsymbol{A_X X_{Bc} E_X}^T + \boldsymbol{E_X X_{Bc} A_X}^T = -\boldsymbol{B_X B_X}^T \tag{25}$$

$$\boldsymbol{A_X}^T \boldsymbol{X_{Bo} E_X} + \boldsymbol{E_X}^T \boldsymbol{X_{Bo} A_X} = (\boldsymbol{E_X}^T \boldsymbol{X_{Bo} B_X} - \boldsymbol{C_X}^T) inv(\boldsymbol{D_X} + \boldsymbol{D_X}^T)(\boldsymbol{C_X}^T - \boldsymbol{E_X}^T \boldsymbol{X_{Bo} B_X})^T = \boldsymbol{0} \tag{26}$$

where:

$$\boldsymbol{X_{Bc}} = \boldsymbol{T_z}^{-1} \boldsymbol{K_c T_z}^{-T} = \boldsymbol{X_B} = diag\ (\psi_1, \psi_2, \ldots, \psi_n) \tag{27}$$

$$\boldsymbol{X_{Bo}} = \boldsymbol{T_z}^T \boldsymbol{J_o T_z} = \boldsymbol{X_B} = diag\ (\psi_1, \psi_2, \ldots, \psi_n) \tag{28}$$

The matrices X_{Bc} and X_{Bo} exhibit diagonal symmetry and positive definiteness, confirming the conditions specified in Theorem 1 and Theorem 2. Therefore, the resulting mixed-balanced system shows stable and passive behavior

Theorem 4: The reduced-order system obtained from Algorithm 1 maintains the stability and passivity of the original system (1).

Proof of Theorem 4. Equations (25) and (26), when represented as matrix blocks, lead to equations (29) and (30).

$$\begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{B1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{x}_{B2} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{11} & \boldsymbol{e}_{12} \\ \boldsymbol{e}_{21} & \boldsymbol{e}_{22} \end{bmatrix}^T + \begin{bmatrix} \boldsymbol{e}_{11} & \boldsymbol{e}_{12} \\ \boldsymbol{e}_{21} & \boldsymbol{e}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{B1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{x}_{B2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} \end{bmatrix}^T = -\begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix}^T \tag{29}$$

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{x}_{B1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{x}_{B2} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{11} & \boldsymbol{e}_{12} \\ \boldsymbol{e}_{21} & \boldsymbol{e}_{22} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_{11} & \boldsymbol{e}_{12} \\ \boldsymbol{e}_{21} & \boldsymbol{e}_{22} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{x}_{B1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{x}_{B2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} \end{bmatrix} \\ &= (\begin{bmatrix} \boldsymbol{e}_{11} & \boldsymbol{e}_{12} \\ \boldsymbol{e}_{21} & \boldsymbol{e}_{22} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{x}_{B1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{x}_{B2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix} - [\boldsymbol{c}_1 \quad \boldsymbol{c}_2]^T) inv(\boldsymbol{D} + \boldsymbol{D}^T) \\ &\times ([\boldsymbol{c}_1 \quad \boldsymbol{c}_2]^T - \begin{bmatrix} \boldsymbol{e}_{11} & \boldsymbol{e}_{12} \\ \boldsymbol{e}_{21} & \boldsymbol{e}_{22} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{x}_{B1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{x}_{B2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix})^T \end{aligned} \tag{30}$$

where:

$$\boldsymbol{x_{B1}} = \boldsymbol{x_{B1}}^{\boldsymbol{T}} = \boldsymbol{diag\ (\psi_1, \psi_2, \ldots, \psi_n)} \tag{31}$$

$$\boldsymbol{x_{B2}} = \boldsymbol{x_{B2}}^{\boldsymbol{T}} = \boldsymbol{diag\ (\psi_1, \psi_2, \ldots, \psi_n)} \tag{32}$$

$$(\boldsymbol{E_r}, \boldsymbol{A_r}, \boldsymbol{B_r}, \boldsymbol{C_r}, \boldsymbol{D_r}) = (\boldsymbol{e_{11}}, \boldsymbol{a_{11}}, \boldsymbol{b_1}, \boldsymbol{c_1}, \boldsymbol{D}) \tag{33}$$

Following this, the reduced-order system satisfies the Lyapunov condition expressed in equation (34).

$$\boldsymbol{a}_{11} \boldsymbol{x}_{B1} \boldsymbol{e}_{11} + \boldsymbol{e}_{11} \boldsymbol{x}_{B1} \boldsymbol{a}_{11}^T = -\boldsymbol{b}_1 \boldsymbol{b}_1^T \tag{34}$$

and the Riccati equation (35).

$$\boldsymbol{a}_{11} \boldsymbol{x}_{B1} \boldsymbol{e}_{11} + \boldsymbol{e}_{11} \boldsymbol{x}_{B1} \boldsymbol{a}_{11}^T = (\boldsymbol{e}_{11}^T \boldsymbol{x}_{B1} \boldsymbol{b}_1 - \boldsymbol{c}_1^T) inv(\boldsymbol{D} + \boldsymbol{D}^T)(\boldsymbol{c}_1^T - \boldsymbol{e}_{11}^T \boldsymbol{x}_{B1} \boldsymbol{b}_1)^T = \boldsymbol{0} \tag{35}$$

x_{B1} is a positive definite, symmetric, and diagonal matrix, meeting the requirements of Remark 1, Theorem 1, and Theorem 2, so the reduced-order system obtained from Algorithm 1 preserves both the stable and passive of the original system (1).

Corollary 2. The system was reduced using Algorithm 1, which employs the controllability Gramian x_{Bc} and the observability Gramian x_{Bo}. Both Gramians exhibit diagonal, symmetric, and positive definite properties, containing *r* Hankel singular values derived from the original mixed-balanced system.

Proof of Corollary 2. From equations (29) to (32), it follows that what must be proven.

Theorem 5. Considering system (1) satisfying Remark 1. When applying the MBT algorithm for order reduction, the upper bound of error is defined by condition (36).

$$\|G_{MBT}(s) - G(s)\|_{H_\infty} \leq 2\sum_{r+1}^{n} \psi_i \tag{36}$$

Proof of Theorem 5. Transforming equation (15) into equation (37)

$$\boldsymbol{A_X}^T \boldsymbol{X_B} \boldsymbol{E_X}, +\boldsymbol{E_X}^T \boldsymbol{X_B} \boldsymbol{A_X} = -\boldsymbol{C}_n^T \boldsymbol{C}_n \tag{37}$$

where $\boldsymbol{C}_n^T$ is as in equation (38)

$$\boldsymbol{C}_n^T = (\boldsymbol{E_X}^T \boldsymbol{X_B} \boldsymbol{B_X} - \boldsymbol{C_X}^T) sqrt(inv(\boldsymbol{D_X} + \boldsymbol{D_X}^T)) \tag{38}$$

System (1) is a mixed-balanced system, and equations (2) and (5) are consequently converted into equations (39) and (40).

$$\boldsymbol{A_X} \boldsymbol{X_B} \boldsymbol{E_X}^T + \boldsymbol{E_X} \boldsymbol{X_B} \boldsymbol{A_X}^T = -\boldsymbol{B_X} \boldsymbol{B_X}^T \tag{39}$$

$$\boldsymbol{A_X}^T \boldsymbol{X_B} \boldsymbol{E_X} + \boldsymbol{E_X}^T \boldsymbol{X_B} \boldsymbol{A_X} = -\boldsymbol{C_n}^T \boldsymbol{C_n} \tag{40}$$

These equations are two Lyapunov equations. Based on the transformations and demonstrations in the BT algorithm, the error according to the H_{inf} norm between the original and reduced-order systems satisfies Theorem 5.

4. Illustrative example

Considering the RLC network as a model of a transmission line (Akram et al., 2020), and selecting the number of nodes $k = 8$, the order of the system is $n = 15$. By convention, the state variables $\boldsymbol{x_{2i-1}}$ represent the voltage across C_i, $\boldsymbol{x_{2j}}$ denotes the current through L_j, u is the input voltage, and y is the output current, where i ranges from 1 to $2k$ and j ranges from 1 to $2k-1$.

We apply the BT, PRBT, and MBT algorithms to reduce the order of the RLC ladder network model from $r = 1$ to $r = n-1$. Figure 1 illustrates the Absolute Error plot using the H-infinity norm between the reduced-order and original systems. Table 1 shows the absolute errors corresponding to each order r of the model.

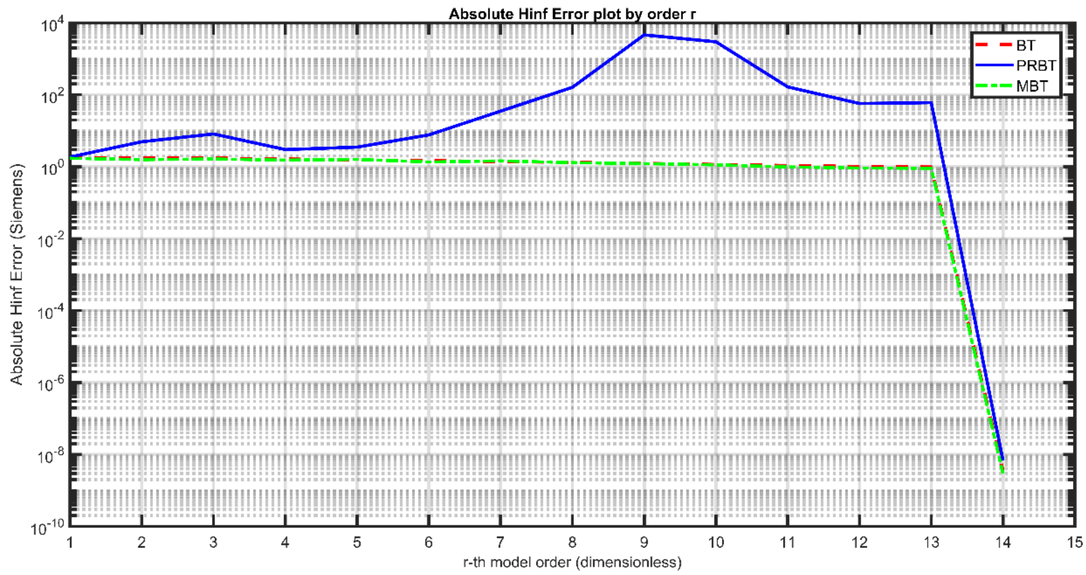

Figure 1 Absolute error plot with decreasing order of r

The plot in Figure 1 depicts the Absolute Hinf Error versus the model order r for three model reduction algorithms: BT, PRBT, and MBT. From this result, we have the following Analysis and Observations:

- MBT algorithm: MBT shows stable and low error values across all model orders. This indicates that MBT is highly reliable and maintains the accuracy of the reduction order model. The error curve for MBT (green dash-dot line) suggests that MBT is an effective method for model reduction.
- BT algorithm: BT's error curve (red dashed line) is stable and does not show significant variations, indicating that low errors are maintained across different model orders. BT, like MBT, proves to be a reliable method for model reduction.
- PRBT algorithm: PRBT shows significant fluctuations in error values across different model orders. As the order decreases, the error of PRBT (represented by the blue solid line) increases rapidly, indicating a lack of stability. The error values vary considerably, depending on the chosen model order. This suggests that PRBT might be less reliable and sensitive to the choice of r.

From Table 1, in conjunction with the numerical results, several insights can be gleaned:

- Both MBT and BT algorithms show consistent and stable error reduction across all model orders. They progressively reduce the *H-infinity* norm error without significant fluctuations, making them both reliable choices for model order reduction.
- While PRBT exhibits large fluctuations in errors, MBT maintains a steady, demonstrating superior stability

The Hankel singular Values (HSV) of the transmission line model, upon transformation into a mixed-balanced system, are detailed in Table 2. In table 2, the HSV gradually decreases as the order r increases. This trend aligns with both theoretical expectations and practical observations, as smaller HSV values indicate a lesser loss of information from the original system, consequently resulting in a proportional increase in the reduction error as the system order decreases.

By comparing Table 2 with the error reduction upper bound formula specified by the MBT method in equation (36), the maximum values of the estimated error are listed in the "Proposed error" column of Table 1. When these predicted errors are compared with the real errors shown in the "MBT error" column of Table 1, it is clear that the formula presented in Theorem 6 is accurate.

Performing order reduction on the RLC ladder network model to achieve a 3rd-order representation, we generate Absolute Error in Amplitude (dB) vs. Frequency (rad/sec), Phase Error (dec) vs. Frequency (rad/sec), and Absolute Error (Amplitude) vs. Time (second) plots for the three methods in Figure 3 and Figure 4, respectively.

Table 1 Comparison of H-infinity Norm Errors between BT, PRBT, and MBT

Model Order (r)	BT Error	PRBT Error	MBT Error	Proposed error
1	1.749991	1.866309	1.717820	13.65631
2	1.734798	4.938759	1.574066	12.31516
3	1.739218	8.155481	1.637961	10.98345
4	1.646226	2.999828	1.525381	9.707632
5	1.534106	3.504173	1.599395	8.461003
6	1.502363	7.664413	1.353905	7.287345
7	1.364971	35.10419	1.451378	6.160561
8	1.328143	161.3269	1.299514	5.111082
9	1.240640	4610.024	1.220345	4.118088
10	1.154465	2971.094	1.127298	3.196535
11	1.083673	165.6717	0.985305	2.329888
12	1.021401	57.23219	0.917855	1.518259
13	0.9848805	60.56017	0.901017	0.945749
14	0	0	0	0

Table 2 The HSV of mixed-balanced system

μ_i	HSV	μ_i	HSV	μ_i	HSV
1	1.226872736948283	6	0.586829045407931	11	0.433323477959059
2	0.670576895359906	7	0.563391797949794	12	0.405814550194131
3	0.665852832023017	8	0.524739486454494	13	0.386254508104516
4	0.637910984013295	9	0.496497172492625	14	0.372874846777396
5	0.623314517294628	10	0.460776660475757	15	0

From the Frequency Domain Error plot in Figure 3, we have the Comparison (Magnitude and Phase Errors) as follows:

- Magnitude Error:

+ error_BT (red dashed line): The magnitude error increases from approximately -20 dB at low frequencies to 0 dB at high frequencies, suggesting that the BT algorithm exhibits higher error compared to both PRBT and MBT.

+ error_PRBT (blue solid line): The magnitude error of PRBT is lower and more stable than BT, but shows slight fluctuations in the mid-frequency range.

+ error_MBT` (green short dashed line): The magnitude error of MBT closely matches PRBT at high and low frequencies, indicating that MBT performs as well or better than PRBT and significantly better than BT.

- Phase Error:

+ error_BT: The phase error for BT is relatively low and stable at low and mid frequencies but increases slightly at high frequencies.

+ error_PRBT: The phase error for PRBT is higher than BT, especially at low and mid frequencies.

+ error_MBT: The phase error for MBT is lower than PRBT and comparable to BT at high frequencies, indicating that MBT maintains better phase accuracy.

- Accuracy in Frequency Domain:

+ BT exhibits larger errors in both magnitude and phase within the frequency domain, in comparison to PRBT and MBT.

+ PRBT improves magnitude accuracy but has higher phase errors.

+ MBT maintains the smallest errors in both magnitude and phase, especially in the high-frequency range, indicating better accuracy in the frequency domain.

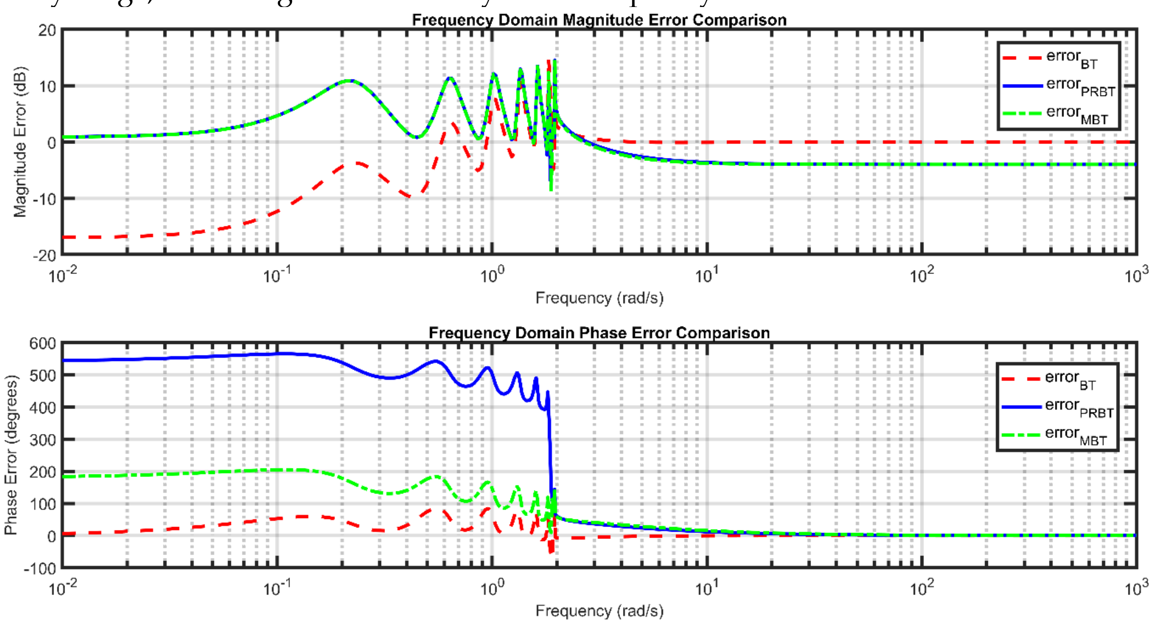

Figure 2 Frequency Domain Error plot between BT, PRBT, and MBT Algorithms

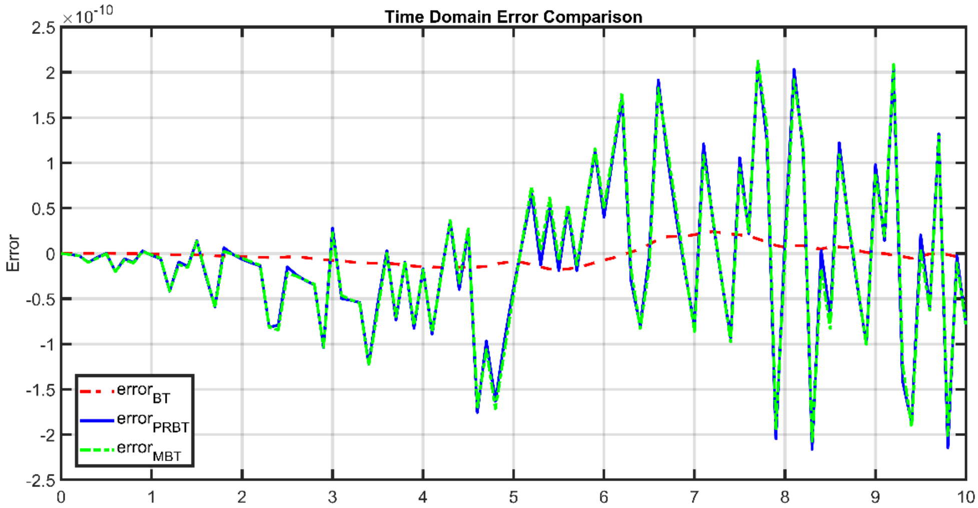

Figure 3 Time Domain Error plot between BT, PRBT, and MBT Algorithms

From The Time Domain Error plot as Figure 4, we have the Comparison between BT, PRBT, and MBT algorithms as follows:

- error_BT: The time domain error for BT is smaller and more stable initially but tends to increase over time.
- error_PRBT: The time domain error for PRBT oscillates around zero, but with larger oscillations than BT.
- error_MBT: The time domain error for MBT is very small and follows PRBT closely, indicating performance that is equivalent to or somewhat better than PRBT but larger than BT.

Overall evaluation:

- MBT proves to be a performing method in both the frequency and time domains, with small and stable errors.
- PRBT is also a good method, particularly for reducing magnitude errors but has higher phase errors.
- BT performs worse compared to PRBT and MBT, with larger errors in the frequency domain.
- Both the MBT and BT algorithms demonstrate high reliability for model order reduction, offering stable and consistent performance across various orders. They both achieve high accuracy at higher orders, making them suitable for practical applications requiring precise reduced models.

Therefore, MBT is the preferred method for minimizing errors and maintaining the highest accuracy in model order reduction systems. MBT proves to be a superior choice when compared to PRBT due to its consistent, low error performance and preserved passivity, while it matches the robustness, reliability, and stability of BT.

5. Discussion and Future Directions

5.1. Contributions to Scientific Theory of the Research Results

The research team developed a model order reduction algorithm (Algorithm 1: Reduce Model Order Using Mixed Balanced Truncation, MBT) to address the limitations of two original algorithms. Specifically, MBT outperforms BT by preserving both stability and passivity, offering lower computational costs and reducing errors compared to PRBT.

During the development of MBT, the authors presented mathematical arguments accompanied by proofs, including Definition 1: Mixed-balanced system definition; Theorem 3: Existence of non-singular transformation; Lemma 1: Eigenvalue invariance under transformation; Lemma 2: Achieving mixed-balance via transformation; Corollary 1: Properties of the mixed-balanced system**;**

Theorem 4: Stability and passivity of reduced-order systems; Corollary 2: Gramian properties in reduced systems; Theorem 5: Error bound in MBT algorithm.

The MBT reduction algorithm simplifies passive circuits with numerous state variables, minimizing computational costs and optimizing the simulation and analysis of high-order systems.

The algorithm and theoretical findings can be applied to the design, development, testing, evaluation, measurement support, response prediction, risk warning, and functional verification of high-order electrical systems using lower-order circuit models.

The content and results of this research can serve as reference material for learning, research, and teaching on system identification, model order reduction, and circuit design and simulation.

The study provides knowledge and source codes to support the development of a reduction toolbox for linear systems in MATLAB.

In resource-constrained environments:

- Complex systems with large, multi-source datasets: Obtaining comprehensive input signals poses challenges in designing, analyzing, surveying, evaluating, predicting, modeling, identifying, and simulating systems. MBT focuses on the most impactful input and output signals identified through preliminary assessments of measurable operational parameters. It does not require a full system model. Instead, it identifies key signals, eliminates less impactful components, and generates a reduced-order model that effectively simulates critical responses without detailed data. This method optimizes resource usage, ensures system performance, and preserves key feedback properties.
- Based on statistical datasets: MBT remains crucial in reducing complexity by focusing on the most significant state variables. This not only simplifies the system but also accelerates signal processing and reduces computational load, ensuring enhanced efficiency while preserving the core dynamic characteristics of the system.

5.2. Computational Costs of the Algorithms

The BT, PRBT, and MBT algorithms rely on the principle of Gramian balancing, followed by truncation of balanced equivalent system matrices. The computational complexity differences stem from solving matrix equations to determine the observability and controllability Gramians of the original system.

Lyapunov equation complexity: $\boldsymbol{O}(\boldsymbol{n}^3 + \boldsymbol{n}\boldsymbol{m}^2)$. If $n \gg m$, the $\boldsymbol{O}(\boldsymbol{n}^3)$ term dominates. If $\boldsymbol{m} \gg \boldsymbol{n}$, the $\boldsymbol{O}(\boldsymbol{n}\boldsymbol{m}^2)$ term dominates. For $\boldsymbol{n} \approx \boldsymbol{m}$, the overall complexity is $\boldsymbol{O}(\boldsymbol{n}^3)$.
Performance Consideration: The complexity is primarily determined by the sizes of n and m. In cases where m is relatively small, the computational cost is dominated by the matrix-matrix multiplications involving **A, P,** and **E.** Solving two Lyapunov equations in BT incurs this cost twice.

Riccati equation complexity: $\boldsymbol{O}(\boldsymbol{n}^3 + \boldsymbol{n}^2\boldsymbol{m} + \boldsymbol{n}\boldsymbol{m}^2 + \boldsymbol{m}^3)$. If $\boldsymbol{n} \gg \boldsymbol{m}$, the complexity is dominated by $\boldsymbol{O}(\boldsymbol{n}^3)$. If $\boldsymbol{m} \gg \boldsymbol{n}$, the complexity is dominated by $\boldsymbol{O}(\boldsymbol{m}^3)$. For $\boldsymbol{n} \approx \boldsymbol{m}$, the overall complexity is $\boldsymbol{O}(\boldsymbol{n}^3)$**.** The inversion of $\boldsymbol{D} + \boldsymbol{D}^T$ and the multiplication involving **B, C** and **D** contribute significantly to the complexity when mmm is large. This is critical for performance optimization. Solving two Riccati equations in PRBT doubles this cost.

In MBT, determining the Gramians involves solving one Lyapunov equation and one Riccati equation, making MBT less computationally expensive than PRBT.

5.3. Limitations of the MBT Algorithm

While MBT achieves better computational efficiency and lower reduction errors than PRBT, its complexity remains higher than BT, with greater deviations from the original model.

As with most algorithms, MBT's accuracy and processing speed depend on factors such as software data types, matrix solver precision, hardware configuration, firmware performance, programming language, implementation optimization, and the complexity of the original system's data.

MBT is designed for linear systems (1) meeting requirements in Remark 1. Systems not satisfying Remark 1 require intermediate transformations to conform to the required format.

Solving complex matrix equations in MBT can lead to increased computational costs for systems with large datasets, posing challenges for hardware with limited processing capabilities.

5.4. *Development Directions*

Reduce algorithm complexity by employing Newton iteration, low-rank approximations, or rational Krylov subspace methods.

To minimize reduction-induced errors, integrate optimization techniques (De Guzman et al., 2024; Jusuf et al. 2024; Wichapa et al. 2024; Nitnara and Tragangoon, 2023; Hendrarini et al., 2022), with objectives such as error minimization and preservation of system physical properties.

For nonlinear systems, preprocess using linearization algorithms before applying MBT.

For unstable systems or those with mixed stable/unstable components:

+ Decompose into stable and unstable subsystems, apply MBT to the stable component, and combine the reduced-order model with the unstable component.

+ Use partial stabilization techniques to transform the unstable system into a stable one before applying MBT.

6. Conclusions

In this paper, we introduced a novel Mixed Balanced Truncation (MBT) algorithm tailored for reducing linear time-invariant continuous-time descriptor systems, specifically within the context of electrical and electronic circuit simulations. Our approach aimed to amalgamate the benefits of Balanced Truncation (BT) and Positive-real balanced truncation (PRBT) while mitigating specific drawbacks. The MBT algorithm demonstrated superior performance in consistently maintaining low error values across various model orders, indicating high reliability for model order reduction with minimal loss of accuracy. Compared to BT and PRBT, MBT showed marked improvement in error metrics, with a steady and predictable decrease in errors, significantly outperforming PRBT, which had large fluctuations and sensitivity to reduction order. MBT retained the essential properties of the original system, including stability and passivity, as confirmed by theoretical proofs and numerical simulations. Its application to an RLC ladder network model effectively reduced computational complexity while preserving the original system's dynamic behavior, which is valuable for resource-limited environments. Additionally, our study contributes to the theoretical understanding of model order reduction by providing new insights into the transformation and balancing of descriptor systems supported by established theorems, lemmas, and corollaries. Future work could explore further optimizations and extensions of the MBT approach to other types of systems and applications, thereby broadening its impact and utility within electronics and electrical engineering.

Acknowledgements

This research was financially supported by the Program of the Ministry of Education and Training of Vietnam under grant number B2023-TNA-17.

Author Contributions

Conceptualization, Methodology, Funding acquisition- Huy-Du Dao(H.-D D); Investigation, Data curation, Validation Ngoc-Kien Vu(N.-K V);Investigation, Validation Van-Ta Hoang (V.-T H); Formal analysis, Software, Resources, Writing—original draft, Writing—review & editing, Thanh-Tung Nguyen (T.-T N); Formal analysis, Supervision, Project administration Hong-Quang Nguyen (H.-Q N). All authors have read and agreed to the published version of the manuscript.

Conflict of Interest

The authors declare no conflicts of interest.

References

Akram, N, Alam, M, Hussain, R, Ali, A, Muhammad, S, Malik, R & Haq, AU 2020, 'Passivity preserving model order reduction using the reduce norm method', *Electronics*, vol. 9(6), p. 964, https://doi.org/10.3390/electronics9060964

Ali, HR, Kunjumuhammed, LP, Pal, BC, Adamczyk, AG & Vershinin, K 2019, 'Model order reduction of wind farms: Linear approach', *IEEE Transactions on Sustainable Energy*, vol. 10(3), pp. 1194-1205, https://doi.org/10.1109/TSTE.2018.2863569

Anaparthi, KK, Chaudhuri, B, Thornhill, NF & Pal, BC 2005, 'Coherency identification in power systems through principal component analysis', *IEEE transactions on power systems*, vol, 20(3), pp. 1658-1660, https://doi.org/10.1109/TPWRS.2005.852092

Antoulas, AC 2005, 'Approximation of large-scale dynamical systems', *Society for Industrial and Applied Mathematics*, https://epubs.siam.org/doi/pdf/10.1137/1.9780898718713.bm

Axelou, O, Floros, G, Evmorfopoulos, N & Stamoulis, G 2023, 'Fast electromigration stress analysis using Low-Rank Balanced Truncation for general interconnect and power grid structures', *Integration*, vol. 89, pp. 197-206, https://doi.org/10.1016/j.vlsi.2022.12.005

Benner, P & Feng, L 2021, 'Model order reduction based on moment-matching', in Benner, P, Grivet-Talocia, S, Quarteroni, A, Rozza, G, Schilders, W & Silveira, LM (eds), *Model order reduction: Volume 1: System-and Data-Driven Methods and Algorithms'*, pp. 57-96, De Gruyter, https://library.oapen.org/bitstream/handle/20.500.12657/52281/1/9783110498967.pdf#page=68

Benner, P & Stykel, T 2017, 'Model order reduction for differential-algebraic equations: a survey, in Ilchmann, A., Reis, T. (eds), *Surveys in Differential-Algebraic Equations IV*, pp. 107-160, Springer, Cham, https://link.springer.com/chapter/10.1007/978-3-319-46618-7_3

Benner, P, Grivet-Talocia, S, Quarteroni, A, Rozza, G, Schilders, W & Silveira LM 2021, '*Model Order Reduction: Volume 2: Snapshot-based methods and algorithms*', pp. 47-96, Berlin, Boston: Walter De Gruyter GmbH & Co KG, https://doi.org/10.1515/9783110671490

Benner, P, Grivet-Talocia, S, Quarteroni, A, Rozza, G, Schilders, W & Silveira, LM 2021, '*Model order reduction: Volume 1: System-and data-driven methods and algorithms*', p. 378, De Gruyter, https://library.oapen.org/handle/20.500.12657/52281

Benner, P, Schilders, W, Grivet-Talocia, S, Quarteroni, A, Rozza, G & Silveira, LM 2020, '*Model order reduction: Volume 2: Snapshot-Based Methods and Algorithms*', p. 348, De Gruyter, https://library.oapen.org/handle/20.500.12657/46696

Breiten, T & Unger, B 2022, 'Passivity preserving model reduction via spectral factorization', *Automatica*, vol. 142, p. 110368, https://doi.org/10.1016/j.automatica.2022.110368

Choupanzadeh, R & Zadehgol, A 2023, 'Blockwise vs. and General MNA for MOR', *2023 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (USNC-URSI)*, pp. 847-848, IEEE, https://doi.org/10.1109/USNC-URSI52151.2023.10237944

De Guzman, CJP, Sorilla, JS, Chua, AY & Chu TSC, 2024, 'Ultra-Wideband Implementation of Object Detection Through Multi-UAV Navigation with Particle Swarm Optimization', *International Journal of Technology*, vol. 15(4), pp. 1026-1036, https://doi.org/10.14716/ijtech.v15i4.5888

Floros, G, Evmorfonoulos, N & Stamoulis, G 2019, 'Efficient Circuit Reduction in Limited Frequency Windows', *2019 16th International Conference on Synthesis, Modeling, Analysis and Simulation Methods and Applications to Circuit Design (SMACD)*, pp. 129-132, IEEE, https://doi.org/10.1109/SMACD.2019.8795231

Fortuna, L, Nunnari, G & Gallo, A 2012, 'Model order reduction techniques with applications in electrical engineering', *Springer Science & Business Media*, https://link.springer.com/book/10.1007/978-1-4471-3198-4

Freund, RW 2000, 'Krylov-subspace methods for reduced-order modeling in circuit simulation', *Journal of Computational and Applied Mathematics*, vol. 123(1-2), pp. 395-421, https://doi.org/10.1016/S0377-0427(00)00396-4

Freund, RW 2022, 'Electronic Circuit Simulation and the Development of New Krylov-Subspace Methods', in *Novel Mathematics Inspired by Industrial Challenges*, pp. 29-55, Cham; Springer International Publishing, https://link.springer.com/chapter/10.1007/978-3-030-96173-2_2

Gräßle, C, Hinze, M & Volkwein, S 2020, 'Model order reduction by proper orthogonal decomposition', https://library.oapen.org/bitstream/handle/20.500.12657/46696/1/9783110671490.pdf#page=56

Grussler, C, Damm, T & Sepulchre, R 2021, 'Balanced truncation of $ k $-positive systems', *IEEE Transactions on Automatic Control*, vol. 67(1), pp. 526-531, https://doi.org/10.1109/TAC.2021.3075319

Gugercin, S & Antoulas, AC 2004, 'A survey of model reduction by balanced truncation and some new results', *International Journal of Control*, vol. 77(8), pp. 748-766, https://doi.org/10.1080/00207170410001713448

Günther, M., Feldmann, U & ter Maten, J 2005, 'Modelling and discretization of circuit problems', *Handbook of numerical analysis*, 13, pp. 523-659, https://doi.org/10.1016/S1570-8659(04)13006-8

Hao, L & Shi, G 2022, 'Realizable reduction of multi-port RCL networks by block elimination', *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol 70(1), pp. 399-412, https://doi.org/10.1109/TCSI.2022.3218548

Hendrarini, N, Asvial, M & Sari, RF 2022, 'Wireless Sensor Networks Optimization with Localization-Based Clustering using Game Theory Algorithm', *International Journal of Technology*, vol. 13(1), pp. 213-224, https://doi.org/10.14716/ijtech.v13i1.4850

Hossain, MS & Trenn, S 2023, 'Midpoint based balanced truncation for switched linear systems with known switching signal', *IEEE Transactions on Automatic Control,* vol. 69(1), pp. 535-542, https://doi.org/10.1109/TAC.2023.3269721

Huisinga, H & Hofmann, L 2018, 'Order reduction in electrical power systems using singular perturbation in different coordinate systems', *COMPEL-The international journal for computation and mathematics in electrical and electronic engineering*, vol. 37(4), pp. 1525-1534, https://doi.org/10.1108/COMPEL-08-2017-0360

Jiang, YL & Xiao, ZH 2015, 'Arnoldi-based model reduction for fractional order linear systems', *International Journal of Systems Science*, vol. 46(8), pp. 1411-1420, https://doi.org/10.1080/00207721.2013.822605

Jiang, YL & Yang, JM 2021, 'Asymptotic waveform evaluation with higher order poles', *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 68(4), pp. 1681-1692, https://doi.org/10.1109/TCSI.2021.3052838

Jusuf, A, Jarwadi, MH, Hastungkorojati, DG, Gunawan, L, Akbar, M, Zakaria, K, Izzaturrahman, MF & Palar, PS 2024, 'Design exploration and optimization of a multi-corner crash box under axial loading via Gaussian process regression', *International Journal of Technology*, vol. 15(6), pp. 1749-1770, https://doi.org/10.14716/ijtech.v15i6.7278

Kassis, MT, Kabir, M, Xiao, YQ & Khazaka, R 2016, 'Passive reduced order macromodeling based on loewner matrix interpolation', *IEEE Transactions on Microwave Theory and Techniques*, vol. 64(8), pp. 2423-2432, https://doi.org/10.1109/TMTT.2016.2586481

Khan, H, Bazaz, MA & Nahvi, SA 2019, 'Singular perturbation‐based model reduction of power electronic circuits', *IET Circuits, Devices & Systems*, vol. 13(4), pp. 471-478, https://doi.org/10.1049/iet-cds.2018.5234

König, J & Freitag, MA 2023, 'Time-Limited Balanced Truncation for Data Assimilation Problems', *Journal of Scientific Computing*, vol. 97(2), p. 47, https://doi.org/10.1007/s10915-023-02358-4

Kumar, R & Ezhilarasi, D 2023a, 'A state-of-the-art survey of model order reduction techniques for large-scale coupled dynamical systems', *International Journal of Dynamics and Control*, vol. 11(2), pp. 900-916, https://doi.org/10.1007/s40435-022-00985-7

Kumar, R & Ezhilarasi, D 2023b, 'A state-of-the-art survey of model order reduction techniques for large-scale coupled dynamical systems', *International Journal of Dynamics and Control*, vol. 11(2), pp. 900-916, https://doi.org/10.1007/s40435-022-00985-7

Lindmark, G & Altafini, C 2017, 'A driver node selection strategy for minimizing the control energy in complex networks', *IFAC-PapersOnLine*, vol. 50(1), pp. 8309-8314, https://doi.org/10.1016/j.ifacol.2017.08.1410

Manthey, R, Knospe, A, Lange, C, Hennig, D & Hurtado, A 2019, 'Reduced order modeling of a natural circulation system by proper orthogonal decomposition', *Progress in Nuclear Energy*, vol. 114, pp. 191-200, https://doi.org/10.1016/j.pnucene.2019.03.010

Mehrmann, V & Stykel, T 2005, 'Balanced truncation model reduction for large-scale systems in descriptor form', In *Dimension Reduction of Large-Scale Systems: Proceedings of a Workshop held in Oberwolfach, Germany*, October 19-25, 2003, Berlin, Heidelberg: Springer Berlin, Heidelberg, pp. 83-115, https://doi.org/10.1007/3-540-27909-1_3

Nitnara, C & Tragangoon, K 2023, 'Simulation-Based Optimization of Injection Molding Process Parameters for Minimizing Warpage by ANN and GA. *International Journal of Technology*. Vol. 14(2), pp. 422-433, https://doi.org/10.14716/ijtech.v14i2.5573

Pavan, S & Temes, GC 2023, 'Reciprocity and inter-reciprocity: A tutorial—Part I: Linear time-invariant networks', *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 70(9), pp. 3413-3421, https://doi.org/10.1109/TCSI.2023.3276700

Phillips, J, Daniel, L & Silveira, LM 2002, 'Guaranteed passive balancing transformations for model order reduction', In *Proceedings of the 39th Annual Design Automation Conference*, pp. 52-57, https://doi.org/10.1145/513918.513933

Poort, L, Besselink, B, Fey, RHB & van de Wouw, N 2023, 'Passivity-preserving, balancing-based model reduction for interconnected systems', *IFAC-PapersOnLine*, vol. 56(2), pp. 4240-4245, https://doi.org/10.1016/j.ifacol.2023.10.1782

Prajapati, AK & Prasad, R 2020, 'A new model order reduction method for the design of compensator by using moment matching algorithm', *Transactions of the Institute of Measurement and Control*, vol. 42(3), pp. 472-484, https://doi.org/10.1177/0142331219874595

Reis, T & Stykel, T 2010, 'Positive real and bounded real balancing for model reduction of descriptor systems', *International Journal of Control*, vol. 83(1), pp. 74-88, https://doi.org/10.1080/00207170903100214

Rydel, M & Stanisławski, R 2018, 'A new frequency weighted Fourier-based method for model order reduction', *Automatica*, vol. 88, pp. 107-112, https://doi.org/10.1016/j.automatica.2017.11.016

Salehi, Z, Karimaghaee, P & Khooban, MH 2021a, 'A new passivity preserving model order reduction method: conic positive real balanced truncation method', *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52(5), pp. 2945-2953, https://doi.org/10.1109/TSMC.2021.3057957

Salehi, Z, Karimaghaee, P & Khooban, MH 2021b 'Mixed positive-bounded balanced truncation', *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 68(7), pp. 2488-2492, https://doi.org/10.1109/TCSII.2021.3053160

Salehi, Z, Karimaghaee, P & Khooban, MH 2021c, 'Model order reduction of positive real systems based on mixed gramian balanced truncation with error bounds', *Circuits, Systems, and Signal Processing*, vol. 40(11), pp. 5309-5327, https://doi.org/10.1007/s00034-021-01734-5

Salehi, Z, Karimaghaee, P, Salehi, S & Khooban, MH 2022, 'Phase Preserving Balanced Truncation for Order Reduction of Positive Real Systems', *Automation*, vol. 3(1), pp. 84-94, https://doi.org/10.3390/automation3010004

Samuel, ER, Knockaert, L & Dhaene, T 2014, 'Matrix-interpolation-based parametric model order reduction for multiconductor transmission lines with delays', *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 62(3), pp. 276-280, https://doi.org/10.1109/TCSII.2014.2368611

Schilders, WHA, Van der Vorst, HA & Rommes, J 2008, Model order reduction: theory, research aspects and applications. Springer, https://link.springer.com/book/10.1007/978-3-540-78841-6

Singh, N, Prasad, R & Gupta, HO 2008, 'Reduction of power system model using balanced realization, Routh and Padé approximation methods', *International Journal of Modelling and Simulation*, vol. 28(1), pp. 57-63, https://doi.org/10.1080/02286203.2008.11442450

Song, QY, Jiang, YL & Xiao, ZH 2017, 'Arnoldi-based model order reduction for linear systems with inhomogeneous initial conditions', *Journal of the Franklin Institute*, vol. 354(18), pp. 8570-8585, https://doi.org/10.1016/j.jfranklin.2017.10.014

Suman, SK & Kumar, A 2021, 'Linear system of order reduction using a modified balanced truncation method', *Circuits, Systems, and Signal Processing*, vol. 40, pp. 2741-2762, https://doi.org/10.1007/s00034-020-01596-3

Tan, S & He, L 2007, 'Advanced model order reduction techniques in VLSI design', *Cambridge University Press*, www.cambridge.org/9780521865814

Unneland, K, Van Dooren, P & Egeland, O 2007a, 'A novel scheme for positive real balanced truncation', in *2007 American Control Conference, IEEE*, pp. 947-952, http://dx.doi.org/10.1109/ACC.2007.4282863

Unneland, K, Van Dooren, P & Egeland, O 2007b, 'New schemes for positive real truncation', *Modeling, Identification and Control*, vol. 28(3), pp. 53-65, https://doi.org/10.4173/mic.2007.3.1

Wichapa, N, Pawaree, N, Nasawat, P, Chourwong, P, Sriburum, A & Khanthirat, W 2024, 'Process of solving multi-response optimization problems using a novel data envelopment analysis variant-Taguchi method', *International Journal of Technology*, vol. 15(6), pp. 2038-2059, https://doi.org/10.14716/ijtech.v15i6.7134

Wittig, T, Munteanu, I, Schuhmann, R & Weiland, T 2002, 'Two-step Lanczos algorithm for model order reduction', *IEEE Transactions on Magnetics*, vol. 38, no. 2, pp. 673-676, https://ieeexplore.ieee.org/abstract/document/996175/

Younes, H, Ibrahim, A, Rizk, M & Valle, M 2021, 'Efficient FPGA implementation of approximate singular value decomposition based on shallow neural networks', *2021 IEEE 3rd International Conference on Artificial Intelligence Circuits and Systems (AICAS)*, pp. 1-4, IEEE, http://dx.doi.org/10.1109/AICAS51828.2021.9458453

Zulfiqar, U, Imran, M, Ghafoor, A & Liaqat, M 2018, 'Time/frequency-limited positive-real truncated balanced realizations', *IMA Journal of mathematical control and information*, vol. 37, no. 1, pp. 64-81, https://doi.org/10.1093/imamci/dny039

Zulfiqar, U, Tariq, W, Li, L & Liaquat, M 2017, 'A passivity-preserving frequency-weighted model order reduction technique', *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 64, no. 11, pp. 1327-1331