



Heuristic Approach to Planning Complex Multi-Stage Production Systems

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Abstract. The paper describes an algorithm for finding a quasi-optimal production plan for complex production systems that involve moving products through a chain of linked processes with varying resource sets. The general problem of optimizing a network of manufacturing enterprises is considered: a set of enterprises producing homogeneous products is modeled during a process consisting of a set of sequential operations arranged in a strict sequence. Standard methods for solving production planning problems are considered. According to the analytical review, most planning tasks in such systems are resolved using original techniques. For this reason, a universal heuristic algorithm was proposed. An algorithm with two branches is also proposed for two cases: for the case of a known constraint in the system and for an alternative case. The algorithm is focused on application in production systems in which the acquisition of empirical data is complicated by the large volume, heterogeneity, and limited reliability of data. In such systems, quasi-optimization in accordance with the proposed algorithm will allow for obtaining a satisfactory result with the permissible and required computing power. The algorithm can be classified as a greedy algorithm. It is partly based on local optimization and performs well for production with a long cycle and a small number of products. For this reason, the approach is recommended for heavy industry, shipbuilding, aircraft manufacturing, and other productions with a long cycle.

Keywords: Greedy algorithm; Heuristic techniques; Optimization algorithms; Production planning; Quasi-optimization

1. Introduction

Selecting the scenario with the best load and resource distribution that satisfies various optimization criteria is necessary to achieve optimal production performance in a complex production system. To tackle the challenge of establishing an optimal production plan, one effective technique is the automation of production schedules. The introduction of ready-made digital solutions for schedule management is an expensive and difficult option in terms of adaptation to the conditions of a particular enterprise. Therefore, it is recommended to develop algorithms and software requirements tailored to optimize the production plan, considering the specific technological processes and resource constraints of the enterprise in question (Zakharenkov, Mrochek, and Mrochek, 2018). Mathematical techniques deployed to tackle uncomplicated tasks like assignments, workshop tasks, or other discrete and combinatorial optimization problems, can be considered conventional means to resolve production planning problems (Lee, Kim, and Kim 2023; Bengio, Lodi,

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and Prouvost, 2021). Such methods include linear optimization methods (Yazdani, Khezri, and Benyoucef, 2021), the Hungarian algorithm (Laha and Gupta, 2016), the branch-and-bound algorithm (Li and Qi, 2022), probability theory and simulation methods (Costas *et al.*, 2023); stochastic optimization methods, for instance, swarm-based algorithms, are applicable to certain specific tasks (Zukhruf, Frazila, and Widhiarso, 2020). However, given the complexity of the problem within the framework presented, none of these methods is appropriate.

This paper examines the general optimization problem for a network of manufacturing enterprises: a set of enterprises producing similar products is modeled during a process consisting of a set of consecutive operations arranged in a strict sequence. The complexity of the task is heightened by diverse resource sets linked to various processes and distinct resource requirements for different products, rendering standard allocation solutions and simple linear optimization algorithms impractical. In addition, when tackling a general task of this nature, it is crucial to take production constraints – procedures that are the chain's most ineffective link – into account. In some practical instances of equivalent systems, the constraints are apparent, while in others, they necessitate an analysis of the production processes.

At the same time, it is evident that the mathematical formalization of production processes simplifies the real system to a certain extent. Hence, the precision of the analytical solution to the optimization issue in this scenario might not satisfy the decision-makers requirements in regard to the balance between the result accuracy and calculation expenses. Therefore, simplifying calculations is an essential task when designing an algorithm to solve an optimization problem. Solutions considering this aspect allow for a quasi-optimal (nearly optimal, although not entirely precise) outcome.

This task is pertinent to enterprises that undertake a few typical but time-consuming orders, specifically in heavy industry, shipbuilding, and aircraft construction.

The aim of this study is to present an algorithm to resolve an optimization issue in a system consisting of orders placed across multiple enterprises and undergoing several production procedures. This algorithm is founded on formulating a general optimization problem and utilizing established mathematical optimization techniques for production planning. The presented method is oriented to problem-solving in the universal case of multistage production, which substantiates the theoretical significance of the study: most of the studied heuristic algorithms (Halim, Hidayat, and Aribowo, 2021; Paramita, Karimah, and Yatmo, 2021; Maulidya *et al.*, 2020) are adapted at the moment to solve specific planning problems.

2. Methods

2.1. Research objective

In general, there are two subtypes of optimization tasks in production systems with multiple sequential processing operations:

1. In the first instance, the system's constraint is known; this paper does not address it because its definition is linked to the techniques of the theory of constraints (Kiran, 2019). In this case, it is essential to consider loading the system constraint as much as possible.

2. In the latter instance, the constraint is uncertain, and thus, it is crucial to consider the potential for its discovery when addressing the issue at hand.

Thus, in the general case, the problem is formally stated as follows: a specific set of orders N (in this scenario, referred to as the "order", which is the entity that passes through the production process from input to output) needs to be optimally assigned to M productions based on a criterion, such as (1).

$$\max(T(N)) \rightarrow \min, \text{ where} \quad (1)$$

$T(N)$ is a set of completion dates for the production processes of the set of N orders. The element of the $T(N)$ set is the completion date of a particular order.

The problem is typically framed as a single-criterion optimization problem. Additionally, other optimization criteria can be introduced, such as maximizing the number of products completed by a given deadline or optimizing the workload of a specific enterprise within the M set. These criteria can either replace or supplement the original criterion to form a multi-criteria optimization task.

The production process comprises P separate procedures that must be executed in a strict sequence. The procedures and their execution sequence remain identical, regardless of the enterprise. In general, transitions between enterprises are not allowed (i.e., it is impossible to perform p_i procedure at one enterprise and p_{i+1} procedure at another one).

Each enterprise in the M set is characterized by a set of R resources having a capacity equal to P . Each enterprise resource is linked one-to-one to the procedure performed at this enterprise.

For each N set order, an S set is created containing the essential resources to perform the procedures with a corresponding P -capacity. Each element in this set is linked one-to-one to the procedure performed for this order.

For each order, an L set is also established to determine the duration of the procedures (the capacity of the set is P). Based on the elements of this set, the objective function $\max(T(N))$ is calculated.

Thus, for the optimization problem, a system of constraints (2) is introduced, the number of inequalities in which is $|M| \times |P|$:

$$\forall m \in M, \forall p \in P: \sum_{i=1}^{|N_m|} S_{ip} \leq R_{mp}, \text{ where} \tag{2}$$

N_m is a subset of the N set: the number of orders executed by the enterprise m ;
 S_{ip} are the resources required to process the order i within the procedure p ;
 R_{mp} are the resources available at the enterprise m to perform the procedure p .

If the system has a known constraint, system (2) takes the form (3):

$$\left\{ \begin{array}{l} \forall m \in M, \forall p \in P \setminus p_0: \sum_{i=1}^{|N_m|} S_{ip} \leq R_{mp}; \\ \forall m \in M: \sum_{i=1}^{|N_m|} S_{ip_0} = R_{mp_0}, \text{ where} \end{array} \right. \tag{3}$$

p_0 - is the procedure that is restrictive in nature, assuming a standardized implementation across all enterprises; any deviations from this standard require adjustments to the system (3).

It is permitted to switch from system (3) to system (2) even in the case of a known system constraint to achieve a quasi-optimal state since it is not always possible to achieve full loading of the constraint. The duration of operations (and the ensuing change in resource requirements) cannot typically be changed to meet the system (3) requirements.

2.2. Existing solutions

In order to formulate the optimization problem in practice and select the optimal equation, the entire production process is taken into account collectively, and each of its sections (procedures) is considered separately (Guseinov, Kurbanov, and Melikov, 2014).

The methods of production process algorithmization are one of the tools for formalizing the system, which enables proceeding to a series of much smaller local subtasks

with reference to the overall goal of system optimization. The process algorithmization requires identifying critical actions that are essential to the overall production process, as well as structuring and decomposing processes, defining required operations, and establishing their relationships with one another (Zhuravleva and Sakovich, 2023; Karimov, 2021).

The algorithm developed under this method for the production process must include a clear sequence of actions and a list of all resources utilized at each stage to ensure the attainment of specified production targets. In developing algorithms for multi-stage processes, the following steps can be distinguished (Dolganov and Zuev, 2015):

1. Technology analysis, decomposition of multi-stage processes, determination of the purpose of each subprocess, its sequence, relationship, and place in achieving the overall goal of the system;

2. Determination of the required resources, encompassing details concerning their characteristics such as the capacity of production sites, equipment accessibility, constraints in processing applications, and additional pertinent factors.

3. Calculation of the size of production sites in relation to the types of products manufactured;

4. Analysis of applications accepted for processing, estimated duration of production (based on historical data on completed projects, resource employment, etc.), required resources, and appropriate production sites;

5. Calculation of the personnel performing basic technological operations.

The system is described as a set of production units (divisions, product processing operations) that are primarily used to analyze resources and their consumption. The chain links are interconnected by technological dependencies on the work complexity and the amount of output (Danilov, Ryzhova, and Voinova, 2010).

Shipbuilding industry optimization can be considered an example of resolving this issue. Such production sites assemble several product units at the same time. When drawing up a production plan, it is challenging to create groups of products from the current order portfolio that are then best assembled simultaneously in the designated order at the company's assembly sites while also taking into account the lowest achievable assembly-related costs (Sidorenko and Khobotov, 2009).

In addition, similar issues can be resolved using information modeling (Lebedeva and Sompol'tseva, 2020). The product information model is the basis for building an automated production management system. Designing databases that contain technological data, including information on necessary equipment and tools, regulatory and technical documentation, and the procedure for technological operations, is essential for building information models. Product information models are used to determine the timing, planning, and need for the purchase of materials. Possible production scenarios are also generated based on the manufactured structure's information model, taking into account the available production capacities (Lebedeva and Sompol'tseva, 2020). It is advisable to use information models of products when creating a production plan to calculate the required resource costs and allocate production areas based on orders. It is possible to identify the optimal strategy for loading the enterprise by comparing data on the specifics of the applications that were received with the current utilization of resources.

Additionally, it is suggested that automated production preparation systems for the shipbuilding industry be introduced. These systems would be based on the division of the hull into blocks and sections and the method chosen for forming the hull on the slipway. A typical technological process of ship assembly is selected for the incoming application on the basis of databases on common sections and blocks (Surkov, 2005).

The analysis of the sources thus reveals that the existing solutions are specific and are applied to particular production conditions or optimization functions. An example of such a solution is a batch scheduling model for a three-stage hybrid Flowshop producing products with a hierarchical assembly structure, applicable to a hierarchical shop floor production structure (Maulidya et al., 2020). A similar example is the development of an algorithm that has as its basis an optimization function for minimizing the total production time (Halim, Hidayat, and Aribowo, 2021). A separate area of research is algorithms for collective production and cooperation (Paramita, Karimah, and Yatmo, 2021), which are not considered in this paper due to their inappropriateness to the task at hand. In this regard, the creation of a single versatile algorithm might be of utmost importance. It also confirms the need for additional algorithmization so that formal methods of task assignment and optimal scheduling are applied adaptively, taking into account the specifics of a particular task.

3. Results and Discussion

The suggested algorithm handles two variations of the general problem (with a known or unknown constraint) by adapting to the conditions. In the case of a known constraint, the optimization model is a two-level one: at the first level, scenarios are generated based on the intention to maximize the load of the constraint procedure; at the second level, the scenarios are refined to form an integral production program. When the constraint is unknown, a quasi-optimal scenario is formed through sequential local optimization. This approach minimizes the complexity of the algorithm implementation. Figure 1 shows the algorithm in broad strokes.

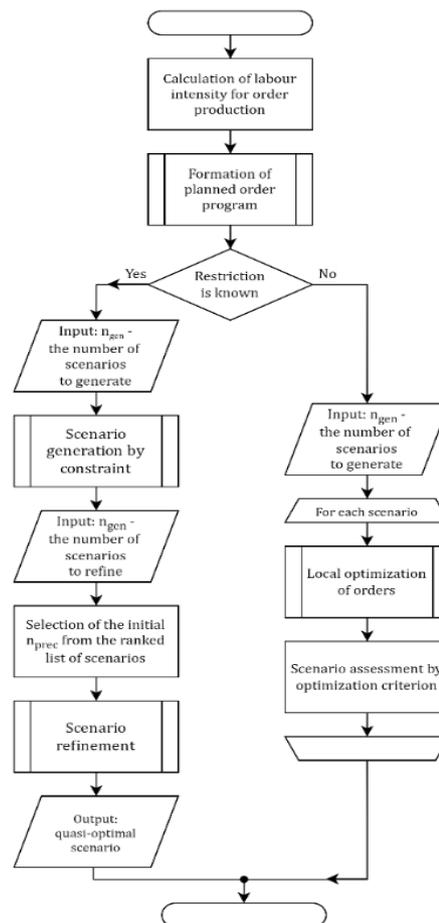


Figure 1 General algorithm for resolving the optimization issue

A two-level algorithm implies the following steps:

1. Make a list of potential order assignment scenarios for enterprises including variations in the processing order and the method used to select a production site for each order. At this point, only the constraint procedure – the system's bottleneck – is taken into account.

2. Rank scenarios for assigning performers in ascending order of the calculated penalty function. When an order is not fulfilled (not included in the scenario), the penalty is assigned. The penalty function can also consider the order priority if there is an order prioritization system. The use of the penalty function is a well-known technique for finding quasi-optimal states within production planning tasks (Laha and Gupta, 2016).

3. Calculate the schedule for all procedures of the first few scenarios with the lowest values of the penalty function. At the same stage, the value of the penalty function's value should be updated subject to the calculated performer schedules by work groups. It is likely that when considering scenarios, the penalty functions will change, and the scenario that was suboptimal at the upper level will become optimal. To achieve this, some scenarios are refined.

4. Select the best scenario based on the penalty function recalculation results.

Figure 2 shows the algorithms for the subprocesses of scenario generation and refinement for a two-level optimization algorithm.

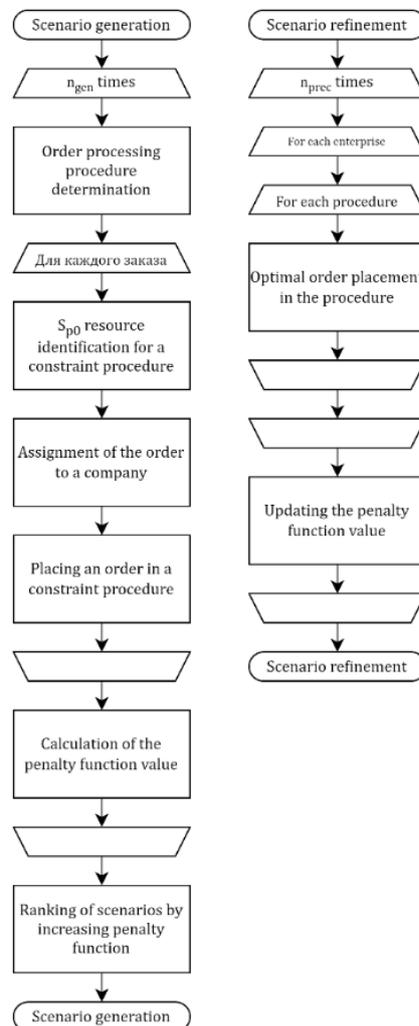


Figure 2 Algorithms for scenario generation and refinement for a two-level optimization model in a task with a known constraint

Let us focus on the areas where it is possible to specify and improve the two-level algorithm. The penalty function may consider additional factors (such as the workload of enterprises, the geographic distance between the business and the customer, etc.) when finalizing and supplementing data. In addition, the order processing procedure and the principle of assigning an enterprise to it can both be decided randomly and based on calculations of the production site's loads, specializations for certain orders, etc.

When a constraint is unknown, a heuristic algorithm for finding a global optimum through local optimization of order placement is adopted. In order to solve these issues, heuristic algorithms are used (Ongcunaruk and Ongcunaruk, 2021; Utama *et al.*, 2019), since this enables obtaining a satisfactory result in the absence of sufficient data and the impossibility of their accurate processing. Low data quality appears likely when taking into account the complexity of the model system under consideration.

A quasi-optimal scenario for the order production in this situation is achieved by generating many different scenarios, both by chance and by some empirical dependence (it is allowed to arrange the processing sequence in ascending or descending order of some order characteristics, for instance, planned labor intensity). The optimization criterion may also take the form of a penalty function akin to the description provided for the two-level branch of the algorithm.

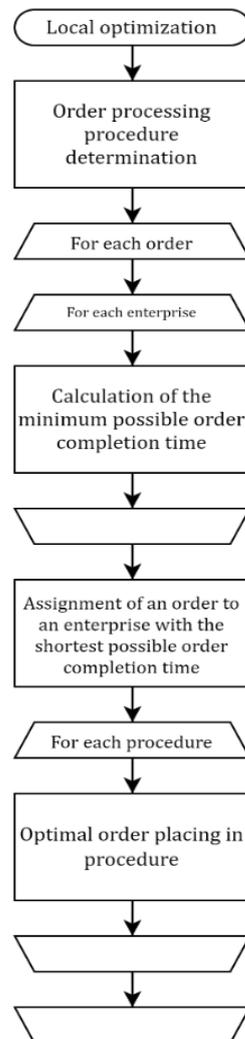


Figure 3 An algorithm for local optimization of order placement for an optimization model in a case with an unknown constraint

Turning to the discussion, we highlight the main features of the algorithm, including its limitations, and describe its primary application areas. We conclude the discussion by noting the main alternative methods for addressing the presented problems.

It should be noted that the algorithm in the second variation, based on local optimization, performs well for production with a long cycle and a small number of products. In the case of a large number of products, the complexity of the greedy algorithm increases significantly, which leads to a nonlinear increase in computational time. For this reason, the approach is recommended for heavy industry, shipbuilding, aircraft manufacturing, and other productions with a long cycle. At the same time, prior research indicates that greedy algorithms for solving such problems are applicable (Wang *et al.*, 2023).

The proposed method is focused on the universal case of multi-stage production, taking into account the principles of the Theory of Constraints, which is an extension and addition to the existing base of methods. Nevertheless, it also has a number of limitations. In particular, it implies a fundamental difficulty in determining the approximation of the quasi-optimal solution to the global optimum, which is a separate future research problem that has not yet been solved in this paper. Another limitation is the applicability of the algorithm to production with a long cycle and a small number of products, which was mentioned above.

The algorithm has undergone testing for optimization issues in a heavy industry enterprise. The results obtained enable the creation of sets of scenarios close to optimal with satisfactory computational time.

In network tasks, a range of optimization methods can be employed to improve efficiency. These include adapted methods for solving assignment problems, workshop tasks, and the branch-and-bound method.

4. Conclusions

Based on the results of the formulation of the production process optimization problem, an algorithm is suggested that can be applied to both problems with known constraints and problems with unknown constraints. The algorithm is focused on quasi-optimization, which limits the effectiveness of its application to the degree of approximation to the globally optimal result. At the same time, it is not possible to reliably assess this degree at the current stage - the development of available methods for such determination is the topic of further research. This particular model for solving production planning issues is appropriate for heavy industry enterprises characterized by parallel and long-term manufacturing of high-tech complex products. According to the analytical review, most planning tasks in such systems are resolved using original techniques. In certain instances, tasks are accomplished using empirical patterns, and the production plan is developed manually. Automating the creation of quasi-optimal scenarios within this industry can prove highly beneficial and efficient, serving as a valuable tool for generating production program scenarios and as a component of decision support systems. The algorithm was tested on the task of compiling a medium-term (5-10 years) production schedule for a heavy industry enterprise. The main limitation in its practical application, except for the above-mentioned limitation of approaching the global optimum, is the speed of computation, which increases strongly with the increase of both production objects and producing subjects or procedures. In this regard, a further research task is to speed up the computation both at the level of the algorithm and in practical program implementation. If this task is solved, it will be possible to talk about the possibility of applying the algorithm in a wider range of tasks, including mass production tasks.

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