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Digital Techniques Share Price Modeling based on a Time-varying Walrasian Equilibrium under Exchange Processes in the Financial Market

Sergey Petrov[1], Sergey Yashin[2], Nadezhda Yashina[3], Oksana Kashina[3*],
Natalia Pronchatova-Rubtsova[3], Valentina Kravchenko[3]

[1]*Department of Financial Management, The Faculty of Economics, National Research University Higher School of Economics, 25/12 Bolshaya Pecherskaya St., 603155, Nizhny Novgorod, Russia*
[2]*Department of Management and Public Administration, The Institute of Economics and Entrepreneurship, Lobachevsky State University of Nizhny Novgorod, 23 Gagarin Ave, 603950, Nizhny Novgorod, Russia*
[3]*Department of Finance and Credit, The Institute of Economics and Entrepreneurship, Lobachevsky State University of Nizhny Novgorod, 23 Gagarin Ave, 603950, Nizhny Novgorod, Russia*

Abstract. This paper aims to develop an analytical theory of share pricing in a financial market environment. The proposed approach corresponds to an actual auction mechanism in which an electronic stock exchange terminal processes real-time data. The theoretical framework is based on the microeconomic model of an individual investor's net demand. Equity resources and resources of "free" capital (exchanged for shares) owned by traders and the investors' perception of the structure of the target portfolio are considered the initial variables of the model. The model differs from the classical theory of asset pricing in its notion of fundamental variables. The relations derived for the aggregated net demand in the stock market describe share pricing as a market exchange that results in the Walrasian equilibrium approximation. The authors offered an appropriate econometric technique for estimating the parameters of instantaneous aggregated net demand. The developed approach was tested using the Walrasian equilibrium concept, which demonstrated that the modeled share price corresponded with the observed share price for the Russian financial market. As an example, the authors presented the results of the investment strategy based on the developed approach for the Russian financial market during the 2008–2009 crash. The authors based their approach on identifying the expectations of the stock market participants through an analysis of high-frequency trading platform information. The microeconomic model describes the motives of traders for placing limit orders in the stock market and associates the price and volume of a particular limit order within the parameters of the capital capacity of the net demand of the trader. The application of the algorithm allows for the monitoring of the financial market situation and reveals the market expectations of traders based on the analysis of information transmitted by an order book of a trading platform.

Keywords: Capital asset pricing; Stock exchange; Stock prices; Trading algorithm; Walrasian equilibrium in the Financial Market

1. Introduction

The need to use digital methods and tools to assess various economic phenomena associated with economic risk and uncertainty has been repeatedly emphasized in several modern papers (Lyukevich et al., 2020; Polyanin et al., 2020; Berawi, 2021). Platforms for online trading have

*Corresponding author's email: oksana_kashina@mail.ru, Tel.: +7-903-059-35-03
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made it possible to monitor the environment of an order-driven market and analyze it in real time. Additionally, they have opened access to valuable information on intra-market processes, which is much richer than commonly used data on prices and trading volumes. The use of this information permits one to track the harbingers and the emergence of new market price trends. In this context, the most important and interesting information, as we see it, pertains to the flows of buy and sell limit orders. Theoretical models, methods, and some results of processing this information can be found in many contributions investigating the market microstructure (O'Hara, 2015; Easley et al., 2016).

There are well-established classical models of capital asset pricing, typically categorized into utility-based and arbitrary-based models (Sharpe, 1964). At the same time, these models used by academics and practitioners have been subject to serious criticism for at least two reasons. First, many authors have noted that the basic assumptions of classical models are not realistic (Dionne and Li, 2011; Zhang and Dong, 2015). Most issues arise from the "self-contradictoriness" of classical models that consider market asset pricing as a solution to a problem of portfolio optimization (in other words—a problem of intertemporal choice) by an "aggregated" investor that is thought of as an assembly of all security holders. Second, classical models do not adequately explain systematic biases, such as stock return excess volatility. Further studies of these effects have led to the appearance of new areas of research in financial economics: behavioral finance (Shiller, 2014), forecasting of stock market volatility (Engle, 2001; Lansing and LeRoy, 2014), and studies of information asymmetry and market manipulation (Aggarwal and Wu, 2006; Zhang and Dong, 2015). Moreover, global financial crisis transmission effects add to the complexity of capital asset pricing models (Mendoza and Quadrini, 2010; Reinhart and Rogoff, 2014). Cochrane (2011) and suggest that refocusing the analysis on prices and long-run payoff streams rather than one-period returns may mitigate the difficulties associated with the classical theory.

Microeconomic models of equilibrium prices for interacting markets are the subject of general equilibrium theory. The Walrasian equilibrium concept (Walras, 1874) has been used in several papers to solve applied asset pricing problems for different markets (Huberman and Stanzl, 2005; Hammond, 2017; Beloso and Garcia, 2020; Paes and Wong, 2020; Ruiter, 2020). Nevertheless, not enough attention has been paid to the possibility of using this concept for predicting prices in the stock market and its practical implementations in the form of investment strategies in the conditions of modern online trading. This is further complicated by the financial market's frequent price changes. In addition, economists have repeatedly faced the problem of designing mechanisms or processes to ensure that Walrasian equilibrium in the market system can actually be achieved (Hammond, 2017). In particular, Huberman and Stanzl (2005) studied the effect of market liquidity on asset pricing. These authors, however, adhered to classical frameworks in which market share prices are established due to shareholders' intertemporal choice. Microeconomic and phenomenological models have explained the structure of the bid-ask spread (Huang and Stoll, 2001) and the influence of investor heterogeneity on market demand and supply fluctuations. They also examined the relationship between the flows of limit orders, bid–ask spread, market depth, and price volatility (Chordia et al., 2000), etc.

Petrov et al. (2013) suggested an alternative version of the Walrasian approach, which focused on share pricing phenomena under exchange among trading participants. The framework does not turn to the principle of intertemporal choice; thus, the condition ofl utility maximization by individuals becomes irrelevant. On the contrary, classical asset pricing theory (Cochrane, 2011) uses other original variables: expected future payoff on shares, statistical characteristics of their random returns, allocation of risky and risk-free assets among shareholders, and the degree of their risk aversion. It must also be noted that, in accordance with the classical point of view, share prices are settled when all investors aspire to optimize their portfolios. In addition, the classical approach posits a time period for investments.

This paper proposes an analytic description of the time-varying Walrasian equilibrium in the stock exchange by constructing a momentary aggregate net demand function. We perform such an experiment in two different ways. First, a direct comparison of momentary calculated share price (estimated per the analytical model) with the observed market price is possible. Second, we explain the observational dependence between stock market demand and supply features and the direction of the ensuing share price trend, which Petrov et al. (2017) previously detected. The method may be considered valid over the long term if active investment strategies based on regularities that the theory provides demonstrate promising results. The authors' approach involves the development of a theory and a practical technique of asset pricing in the financial market, which corresponds to the real mechanisms of an auction in the stock exchange's electronic terminal. This allows for the real-time processing of stock data and allows the prediction of prices and construction of investment strategies in comparison with other approaches based on the concept of Walrasian equilibrium.

2. Methods

Suppose that an economy includes shares of N issuers that are traded in a stock market. Evidently, orders to buy appear when investors aim to exchange "cash" for stock; in contrast, orders to sell reflect investors' seeking to exchange stock for cash.

Consider an investor who trades the i^{th} share $(i = 1,..,N)$ at some arbitrarily selected time t (we denote their number by "k"); in general, an investor's portfolio includes either a variety of different shares or cash. More precisely, we should consider a small time period $(t;\ t + dt)$; we imply that the investor sends their orders to buy or sell within this period. Let $n_i^{(k)}$ be the investor's current resource of the i^{th} share; $M^{(k)}$ is their current cash resource, p_i is the current price for the share "i". Hence, the total wealth $W^{(k)}$ of the investor under examination can be represented as follows

$$W^{(k)} = M^{(k)} + \sum_{j=1}^{N} p_j n_j^{(k)} \tag{1}$$

We mark the security under consideration with the index "i"; by contrast, the index "j" is used for summing over all types of assets.

An investor's efforts to trade stock signify that they are not satisfied with their portfolio structure. Let us assume that this investor's aim is to have the relative amount of the i^{th} share in their portfolio at the same moment be equal to $x_i^{(k)}$ within the limits of budget constraint. We do not consider the motives for such an investor's decision; it is taken as given. Indeed, we do not explicitly associate it with the notion of an optimal portfolio structure. Likewise, we do not discuss the motives that may influence the investor to trade stock or abstain from trading; these motives can be caused by economic forces or other reasons. The activity of other traders can also affect investor sentiment. In this case, an appropriate resource of the share $\tilde{n}_i^{(k)}$ that is interpreted as the investor's "demand to hold" may be evaluated from the relation

$$p_i \tilde{n}_i^{(k)} = x_i^{(k)} \left(M^{(k)} + \sum_{j=1}^{N} p_j n_j^{(k)} \right) \tag{2}$$

where all share prices p_j are exogenous variables.

The instantaneous "net demand" determines the investor's position in market trading:

$$\delta n_i^{(k)} = \tilde{n}_i^{(k)} - n_i^{(k)} \tag{3}$$

which represents the amount of the i^{th} share the investor wishes to buy at this price at the time considered; the negative values of $\delta n_i^{(k)}$ indicate that the investor is inclined to sell. We can then express the investor's net demand ("individual net demand") using relation (2) and by isolating the term with $j = i$ under the summation sign in the relation (2); in such a manner, we obtain

$$\delta n_i^{(k)} = \frac{\delta F_i^{(k)}}{p_i} - \delta C_i^{(k)} \tag{4}$$

where the following notations were introduced:

$$\delta F_i^{(k)} = x_i^{(k)} \cdot \left(M^{(k)} + \sum_{\substack{j=1 \\ j \neq i}}^{N} p_j \, n_j^{(k)} \right) \tag{5}$$

$$\delta C_i^{(k)} = n_i^{(k)} (1 - x_i^{(k)}) \tag{6}$$

The summing in expression (5) includes all securities *j* except for the selected share *i*. In general, the parameter $x_i^{(k)}$ characterizing the investor's expectations and appearing in the relations (5) and (6), depends on the share price p_i. However, we will not consider this dependence in the sequel (we assume it produces a slight change in individual net demand function (4) in the short run); in such a manner, the combinations $\delta F_i^{(k)}$ and $\delta C_i^{(k)}$ become independent of the share price p_i also. Expressions (4), (5), and (6) relate to the individual stockholder's current market position, their resources of risky assets, the money source, and their notion of the preferred portfolio structure, respectively. Next, a consistent theory of the time-varying Walrasian equilibrium in the stock market requires aggregating instantaneous net demand functions for all stock-trading participants. To model the asset pricing phenomena, it is convenient to express the aggregate net demand function as a sum of appropriate functions for "buyers" $\Delta n_i{}^+$ and "sellers" $\Delta n_i{}^-$:

$$\Delta n_i = \sum_k^{\substack{over\ all \\ participants}} \delta n_i^{(k)} = \Delta n_i{}^+ + \Delta n_i{}^- \tag{7}$$

$$\Delta n_i{}^+ = \sum_k^{\substack{over\ the\ group \\ of\ buyers}} \delta n_i^{(k)} = \frac{F_i{}^+}{p_i} - C_i{}^+ \tag{8}$$

$$\Delta n_i{}^- = \sum_k^{\substack{over\ the\ group \\ of\ sellers}} \delta n_i^{(k)} = \frac{F_i{}^-}{p_i} - C_i{}^- \tag{9}$$

We use "buyers" and "sellers" to refer to the participants placing orders to buy or sell, respectively. The parameters $F_i{}^+$ and $C_i{}^+$ for buyers, as well as $F_i{}^-$ and $C_i{}^-$ for sellers, in instantaneous aggregate net demand functions (8), (9) are defined by summing the current microscopic characteristics $\delta F_i^{(k)}$ and $\delta C_i^{(k)}$ over the appropriate groups of traders.

$$F_i{}^+ = \sum_k^{\substack{over\ the\ group \\ of\ buyers}} x_i{}^{(k)} * \left(M^{(k)} + \sum_{\substack{j=1 \\ j \neq i}}^{N} p_j \, n_j{}^{(k)} \right) \tag{10}$$

$$C_i{}^+ = \sum_k^{\substack{over\ the\ group \\ of\ buyers}} n_i{}^{(k)} (1 - x_i{}^{(k)}) \tag{11}$$

$$F_i{}^- = \sum_k^{\substack{over\ the\ group \\ of\ sellers}} x_i{}^{(k)} * \left(M^{(k)} + \sum_{\substack{j=1 \\ j \neq i}}^{N} p_j \, n_j{}^{(k)} \right) \tag{12}$$

$$C_i{}^- = \sum_k^{\substack{over\ the\ group \\ of\ sellers}} n_i{}^{(k)} (1 - x_i{}^{(k)}) \tag{13}$$

Notations *F* and *C* for the appropriate parameters are concerned with the terms "effective free capital" and "effective capacity" (Petrov et al., 2013; Petrov et al., 2017).

Note that the function $\Delta n_i{}^+$ models price dependence of demand in the range of its positive values $\Delta n_i^+ > 0$; likewise, model price dependence of supply is characterized by the branch of the function $\Delta n_i{}^-$ situated in its negative region $\Delta n_i^+ < 0$. Equations 7–9 for instantaneous aggregate net demand functions and representations (10)–(13) for combinations $F_i{}^+$, $C_i{}^+$, $F_i{}^-$, $C_i{}^-$ provide for the analytical description of share prices under stock exchange trading. It is evident that the Walrasian equilibrium will be observed in the stock market for i^{th}-type of shares if their price p_i sets the function (7) to zero. Using a clear representation of the instantaneous aggregate net demand function

$$\Delta n_i = \frac{F_i}{p_i} - C_i \tag{14}$$

one can immediately obtain for the equilibrium price

$$P_i = \frac{F_i}{C_i} = \frac{{F_i}^{+} + {F_i}^{-}}{{C_i}^{+} + {C_i}^{-}} \tag{15}$$

The Walrasian approximation is quite coherent with the actual auction pricing mechanism of marketplace trading, wherein share prices equalize the opposite flows of orders to buy and sell. The momentary share price P_i in the equilibrium is a direct function of the "free capital" of shareholders who trade the i^{th} shares at some instant. This "free capital" includes either cash or wealth invested in other shares. The relations explain the well-known effect that the prices of various stocks interact considerably, and that their returns usually have a positive correlation. The weight multiplier $(1 - x_i^{(k)})$ in relations (11) and (13) for parameters ${\text{C}_i}^{+}$ and ${\text{C}_i}^{-}$ reflects the fact that shares become "rare" (Walras, 1874) if investors are more inclined to hold them.

The relations (10)–(13) and (15) represent the equilibrium share price through the variables characterizing demand and supply under exchange processes in the stock exchange. The dynamics of the variables are the result of variations either in market activity and the qualitative structure of traders or in their expectations. These dynamics are evidently affected by several fundamental factors (e.g., economic conditions, news of commodities and financial markets, etc.) and other motives of investors that have a significant and ambiguous effect on their trading activity.

Another interpretation of asset pricing phenomena is that it is radically different from the concept of classical financial economics. Share pricing (see Equation 15) is not associated with the problem of investors' intertemporal choice and with the discounting procedure. The model Equations 10–15 do not use key variables of the "two-parameter world of Sharp and Lintner," (Fama, 1970) as neither the ex-ante covariance of returns nor the parameters characterize the degree of an investor's risk aversion. The developed concept of market equilibrium also does not appeal to the idea of market portfolio optimization given that classical theory considers market portfolios to be widely diversified, including all financial assets (Roll, 1977). Next, using the relation (15) for all shares $(i = 1, .., N)$ traded on the stock exchange and performing some manipulations, we can obtain the system of N linear stochastic algebraic equations for equilibrium share prices P_i:

$$\left(\sum_k^{\substack{over\ all \\ participants}} n_i^{(k)}(1 - x_i^{(k)})\right) \cdot P_i - \sum_{\substack{j=1 \\ j \neq i}}^{N} \left(\sum_k^{\substack{over\ all \\ participants}} x_i^{(k)} n_j^{(k)}\right) \cdot P_j = \sum_k^{\substack{over\ all \\ participants}} x_i^{(k)} M^{(k)} \tag{16}$$

The Walrasian equilibrium approximation permits modeling the asset pricing phenomena using algebraic equations (avoiding modeling transients). The dynamics of the coefficients and the right-hand sides of Equation 16 have a chaotic character. The reason for this is that an assembly of stock market traders changes all the time (new traders with their preferred relative amounts of assets $x_i^{(k)}$, current resources $n_i^{(k)}$ and $M^{(k)}$ come into the market; other traders close their positions). In such a manner, random walks (along with regular changes) are transmitted to market share prices. Therefore, the developed approach to share pricing phenomena under exchange processes focuses on the market prices of securities but not on their expected returns (in contrast to classical models). This concept agrees with the prospects for the development of the asset pricing theory forecasted by Cochrane (2011). It is important to note that a formal solution of Equation 16 makes it possible, in principle, to carry out a consistent microeconomic study of fluctuations of return on assets and their statistical characteristics. At the same time, the classical asset pricing theory considers the covariance of random return as exogenous variables.

The developed microeconomic approach to asset pricing under exchange processes in the stock market opens the door to multiple opportunities for future research. A fundamental question for evaluating the theoretical and practical significance of the developed model is its ability to explain the long-term dynamics of average share prices (i.e., to analyze and interpret historical average return on assets). Similar investigations would permit a comparison of the key results of the model with the conclusions of the classical asset pricing theory.

In conclusion, let us consider a geometric illustration of the time-varying Walrasian equilibrium in the stock market that permits us to imagine the meaning of the key parameters of the suggested model more vividly. The hyperbolic curve in Figure 1 marks the model dependence (equation (14)) between the aggregate net demand Δn_i and the share price p_i. The "effective capacity" of the stock market characterized by the parameter C_i (interval OH on Figure 1) determines the left shift of the hyperbolic curve horizontally (the dashed vertical line is the asymptote to the hyperbolic curve). The dashed oblique line is the tangent line to the hyperbolic curve at the point of equilibrium ($\Delta n_i = 0$). Calculating the slope of the tangent to the X-axis based on the expression (14) and using a simple algebraic manipulation, one may see that the value of the parameter F_i ("effective free capital" of the stock market) defines areas of similar HKL and OP$_i$L triangles. Their areas are equal to $2F_i$ and $F_i/2$, respectively.

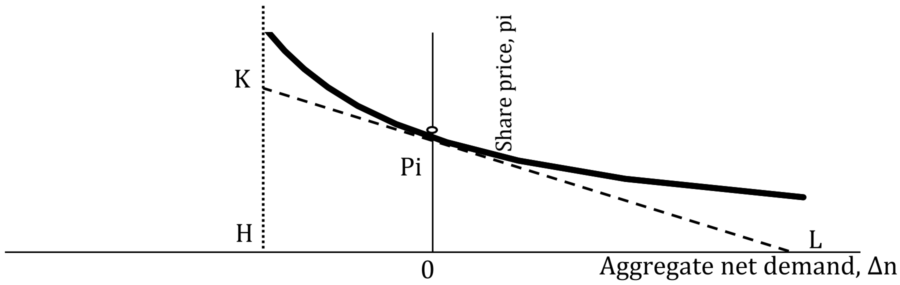

Figure 1 Price dependence of aggregate net demand in the stock exchange and geometric meaning of Walrasian equilibrium's parameters

Hence, if the value of "effective free capital" F_i (i.e. an area of the HKL triangle) is fixed, the form of the triangle (the equilibrium price P_i and the slope of the tangent to the X-axis also) will vary depending on the effective capacity of the stock market C_i. On the practical side, this conclusion essentially explains a well-known fact that the equilibrium is not stable enough in a "thin" market.

3. Results and Discussion

The modern trading system of a stock exchange permits the observation of instantaneous pictures of demand and supply in real time. This assertion is correct, at least for a set of limit orders. Thus, a share pricing model using the Walrasian concept of time-varying stock market equilibrium poses considerable practical interest. The exchange trading platform continuously transmits an ordered array of the best buy and sell limit orders ("trade blotter"). In this manner, we can momentarily plot the stepped curves of demand and supply.

We consider simpler algorithms for the verification of our approach to asset pricing that was developed and tested by the authors for several Russian shares over a limited period from 2008 to 2019.

Figure 2 demonstrates an example of such curves for the shares of the Gazprom, a Russian corporation. The right-hand and left-hand sides of the stepped curve show sets of the best buy and sell limit orders (10 limit orders in each set), respectively. Approximating both parts of the curve by model functions (8) and (9), one may calculate parameters F_i^+, C_i^- and F_i^-, C_i^- using econometric methods. When applying the conventional linear regression technique, there are good reasons to consider instantaneous demand and supply as functions of the additional variable p_i^{-1}, which is inverse to the share price p_i. In the case in hand, fitted dependences (8) and (9) of demand and supply are shown in Figure 2 as parts of the hyperbola lines.

The automated daily procedure of periodic online registration of offsetting limit orders for several common stocks in the Moscow Exchange every two minutes, synchronous with their current prices, was organized. Momentary values of “effective resources of free capital” ${F_i}^+$, ${F_i}^-$ and "effective resources of shares" ${C_i}^+$, ${C_i}^-$ are estimated for every record of bid-ask quotations, and they vary considerably from one record to another (Figure 3).

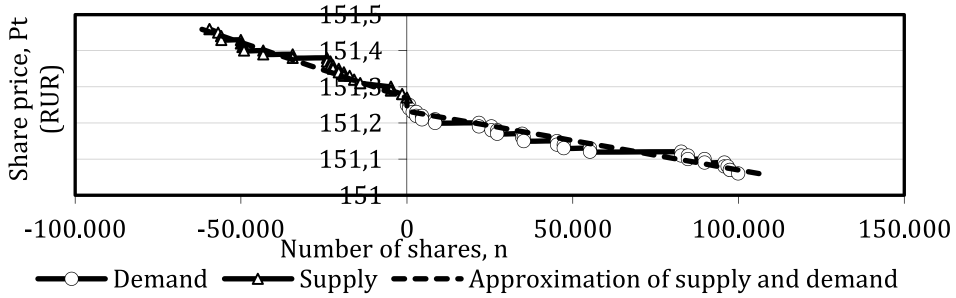

Figure 2 Observed and modeled demand and supply dependences (for the set of limit orders) for the shares of Gazprom Corporation 29.03.2019 at 10.58.35

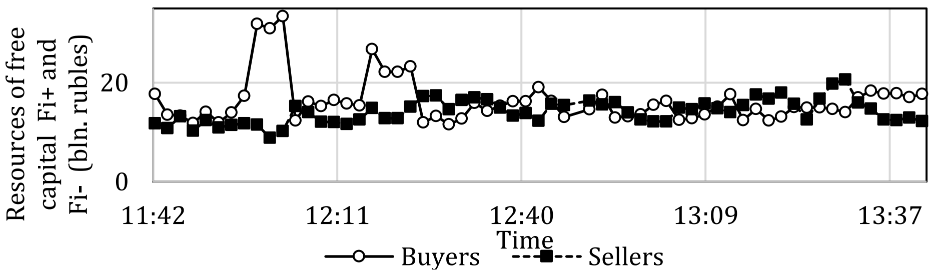

Figure 3 Dynamics of “effective resources of free capital” ${F_i}^+$ and ${F_i}^-$ for the shares of Gazprom Corporation, 29.03.2019 (between 11.43 and 13.43)

Figure 3 demonstrates the momentary values of parameters ${F_i}^+$, ${F_i}^-$ for shares of Gazprom Corporation within a trading session conducted on March 29, 2019, a randomly chosen date.

Table 1 shows that the observed and modeled share prices are closely linked (R^2-statistics is greater than 0.99). This result is remarkable because considerable errors are unavoidable if we model instantaneous demand and supply using only 10 limit orders. Next, to conclude that the model of asset pricing in the short run is correct upon the average, we analyze parameters a_i and b_i of the equation of linear regression

$$p_i^{observed} = a_i + b_i \cdot P_i^{modeled} \tag{17}$$

The random nature of limit order placing is reflected in fluctuations in the modeled share price. If a slope coefficient $b_i$is close to 1 and a shift a_i is about 0, we can expect that the time-varying Walrasian equilibrium approach adequately describes market share pricing phenomena.

We have conducted the study based on a regression estimation (17) for several Russian stocks; this permits us to elucidate how the modeled share price relates to the observed share price for various securities. Table 1 reflects some results obtained on November 21, 2017 for the shares of Sberbank of Russia, LUKOIL, Gazprom, and Surgutneftegas.

Table 1 Results of the regression model (17) for several Russian shares, 21.11.2017 (fragment)

Criteria	Sberbank of Russia Shares	LUKOIL Shares	Gazprom Shares	Surgutneftegas Shares
Slope coefficient b_i ,	0.984	0.999	0.993	1.015
Its 95% confidence interval	0.968…1.001	0.989…1.011	0.986…1.000	0.992…1.038
t-statistics for b_i	118.81	184.54	285.86	86.3
Shift a_i (rubles) ,	1.606	0.92	0.918	-0.435
Its 95% confidence interval	-0.081…3.292	-20.02…21.85	0,020…1.815	-1.128…0.258
t-statistics for a_i	1.88	0.09	2.02	-1.24
Adjusted R2-statistics	0.9904	0.9959	0.9983	0.9819
Number of observations	139	139	139	139

Table 1 makes it clear that the target value of the slope coefficient $b_i = 1$ falls into its 95% confidence interval for all securities under study; additionally, the student's t-test confirms for sure that the result is statistically significant. The regression line's shift a_i is in accordance with its target value (zero) for three securities, as shown in Table 1; the confidence interval a_i does not cover the zero value only for shares of Gazprom Corporation. However, the difference is comparatively small (the shift a_i is about 0.8% of the share price) and rather accidental, and the carried out analysis permits considering it as the uncertainty of measurement that marks the particular trading day.

Summarizing the study's conclusions, we infer that the Walrasian equilibrium approximation is quite acceptable to model instantaneous share prices for the most liquid Russian securities. However, an agreement between the modeled and observed share prices is worse for less liquid securities; this effect is seen in Table 1 for shares of Surgutneftegas Corporation, whose trade volume is considerably less than that of the other three assets.

Microeconomic representations (10) and (12) as well as (11) and (13) permit the detection of changes in the "market sentiment" either for owners of "effective free capital" or for shareholders of the selected type of shares, respectively. For instance, variations in the total activity of capital owners inform the stability of market conditions.

Our studies have shown that it is possible to forecast the share price trend's breakpoints if we analyze the dynamics of the parameters F_i^+, F_i^-, C_i^+, C_i^-, and changes in market share price. In many circumstances, the indicators cause the signals to go long just before the uptrend starts and close a position if the downtrend is expected to come in the short run, for example, if parameters F_i^+ and F_i^- over some time show that the "free capital" of demand-side predominates systematically (Petrov et al., 2017). Based on these indicators of the share price trend's breakpoints, the authors constructed and tested several active portfolio strategies adaptable to an investor's preferences relating to return and risk (potential drawdown). An empirical drawdown criterion may be preferable for active portfolio management over the academic criterion of the standard deviation of return. These strategies permit us to take capital out of the shares if a share price decline is expected and go long on the stock exchange if a rally is predicted.

We verified all the strategies for some securities and for different time frames.

The developed procedure ensured the high efficiency of portfolio investments (Petrov et al. 2017), in particular, during the 2008–2009 financial crisis, wherein share price variations (for examples, for the shares of Sberbank of Russia, LUKOIL) increased by a magnitude of 4–5. For example, Table 2 illustrates the results of such an active portfolio strategy (i.e., its growth and drawdown of capital).

Table 2 Comparing results of passive investment strategy and active investment strategy based on the Walrasian approach to capital asset pricing phenomena

Investment Criteria	Passive investment strategy			Active investment strategy		
	Sberbank of Russia Corporation Shares	LUCOIL Corporation Shares	Gazprom Corporation Shares	Sberbank of Russia Corporation Shares	LUCOIL Corporation Shares	Gazprom Corporation Shares
Capital growth, %	-53.8	-11.6	-42.9	37.5	61.0	49.0
Capital drawdown, %	84.2	71.7	76.0	36.9	14.1	19.5

Table 2 demonstrates, in particular, that the investor who used a simple passive strategy ("buy and hold") might have lost nearly 84 percent of their wealth on Sberbank of Russia's shares. At the same time, the strategy based on the approach described above permitted reducing the drawdown to 37 percent. Similar results were obtained for other securities. By applying the developed technique to continuous or periodic regimes, one can track the dynamics of investor sentiments. Digital methods for diagnostics of the behavior of market players have another exciting application. The movement of capital between the demand and supply quotes in an order book should signal insiders' activity and the market manipulation phenomena; previously, similar signals were detected by Petrov et al. (2017) using much simpler procedures. The ideas and methods developed in the present paper allow one to construct sensitive indicators of capital movements. By using these indicators in an automated mode, one can significantly increase the efficiency of countering illegal trade practices by national regulator of financial markets.

4. Conclusions

We propose a model that describes the share pricing phenomena under stock exchange trading using the Walrasian concept of momentary market equilibrium. The model is substantially different from the classical theory of asset pricing in its notion of the fundamental variables responsible for share evaluation. In the classical model, every individual solves the problem of intertemporal choice; in contrast, our model posits an equality of simultaneous opposite flows of orders to buy and sell. It is not an optimization problem per se; there is no objective function for the problem of exchange (Neumann and Morgenstern, 2007). Finally, in describing the market exchange phenomena, the suggested model operates directly with share prices; in contrast, the classical approach typically evaluates the expected return on an asset.

We have developed a technique for verifying the model; its test results on highly liquid Russian shares demonstrates that share price modeling per the Walrasian equilibrium concept is in good agreement with the observed share prices in the stock exchange. However, the closeness of agreement of the suggested model decreases slightly if we consider less liquid shares. In a practical sense, the model opens the way for online diagnostics of the behavior of investors who trade stocks based on information transmitted by trading terminals. Our experimental procedure permits the analysis of the dynamics of the trading behavior of either "free capital holders" or shareholders on the demand and supply sides. The dynamic behavior that we uncover may reflect insiders' activity in the stock exchange. Active portfolio strategies monitoring the activity of large investors using the model may yield significant returns.

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