Inventory Ship Routing and Cargo Stowage Planning on Chemical Tankers

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Abstract. Chemical tankers are a type of ocean carrier with multi-compartments to simultaneously carry various liquid chemicals in bulk and prevent their mixing. This paper discusses the difficulties chemical tanker managers experience when planning vessel routes and scheduling inventory maintenance because of chemical tankers’ unique characteristics and operational constraints. To date, no models have addressed chemical tankers’ inventory routing and scheduling needs while accounting for these challenges. Bridging the research gap, we propose a novel, integrated, mathematical model of inventory ship routing and stowage planning problem (ISRSPP) for chemical tankers. We seek to combine stowage planning, which is an operational problem, with inventory ship routing, which is a tactical problem, through integrated tactical planning. Our objective is to propose a solution with minimal total voyage costs. For this purpose, we formulate our problem in a mixed integer linear program. We build two scenarios to analyze the models applicability and performance, and we solve both of them using a commercial solver. Our results confirm that the stowage planning problem cannot be separated from the inventory ship routing problem for chemical tankers because such a separation could lead to fleet routes for which no feasible stowage plan is possible.

Keywords: Chemical tankers; Integrated planning model; Inventory ship routing; Mixed-integer linear programming; Stowage planning

1. Introduction

Maritime transport, an important pillar of world trade and globalization, is critical for economic growth and sustainable development (Akbulaev & Bayramli, 2020; Bagoulla and Guillotreau, 2020; Dui et al., 2021). Disruption due to the coronavirus disease 2019 (COVID-19) pandemic, such as a short-term decrease in the volume of global maritime trade by 4.1% in 2020 (UNCTAD, 2020), presents challenges and opportunities to build resilience and sustainability in the maritime transport sector (Dulebenets, 2019; Dui et al., 2021; Berawi et al., 2020). This turmoil has also influenced the US$5.7 trillion global chemical industry, which has been an integral part of the global economic landscape for many years. Nevertheless, reaching over US$2.9 trillion in 2019, the global chemical trade has realized moderate average annualized growth of 4.54% over the last decade (World Trade Organization, 2020). As a result, the global demand for commercial shipping fleets

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—including chemical tankers—has remained strong. In 2019–2020, chemical tankers’ dead-weight capacity tonnage grew by 2.9% \((\text{UNCTAD, 2020})\).

In this paper, we present an integrated, mathematical model of chemical tankers’ inventory ship routing and stowage planning problem (ISRPP). Chemical tankers can be distinguished from other ocean bulk carriers by their multiple compartments to simultaneously store various liquid chemicals in bulk and prevent their mixing. Parcel tankers’ compartments are equipped with separate cargo pumping systems. Each cargo pumping system features one hydraulically driven, submerged cargo pump with independent piping, which enables the simultaneous handling of multiple cargoes without mixing.

Chemical tanker managers must address two compatibility constraints for safety concerns regulated by the International Maritime Organization \((\text{Oh & Karimi, 2008})\). According to the International Bulk Chemical (IBC) Code, first, both the construction and coating materials of a compartment determine the chemical cargoes that can be loaded. Typically, chemical tankers with stainless steel compartments can carry a wider range of chemical cargoes than those with compartments lined with organic, epoxy-based and inorganic, zinc silicate-based coating materials \((\text{Neo et al., 2006; Oh & Karimi, 2008})\). Second, the cargoes loaded into chemical tankers’ adjacent compartments must be non-reactive. If the common bulkhead of neighboring compartments is cracked, incompatible cargoes can create a disastrous chemical reaction. The US Coast Guard Compatibility Chart specifies a related regulatory stowage restriction.

To ensure vessel stability, two requirements must be fulfilled. First, vessels must comply with intact \((\text{Marine Safety Committee, 2008a})\) and damage stability \((\text{Marine Safety Committee, 2008b})\) requirements. Operationally, these requirements are met by determining how high a vessel’s center of gravity is in loading conditions, which is a function of the vessel’s draft and accounts for the free-surface effect \((\text{Øvstebø et al., 2011})\). Second, a vessel must ensure its stability during a voyage by properly distributing cargoes’ weight across compartments so that it does not trim excessively. Trim by either the bow or the stern must be limited, depending on the vessel’s design. Additionally, the allocation of ballast water into ballast tanks is also important to maintain a vessel’s stability \((\text{Zeng et al., 2010; Braidotti et al., 2018})\). However, this problem must be addressed by tanker operators anticipating uncertainty in a dynamic operational environment.

Next, during loading and unloading activities, a vessel’s structure must be strong enough to withstand unevenly distributed weight. At a given draft and trim in still water, buoyant force is also non-uniformly distributed along a vessel’s length, though in a fixed fashion since each unit length of the vessel experiences a downward force equal to the weight of water displaced by a transverse section of the corresponding unit length \((\text{Eyres & Bruce, 2012})\). Therefore, either an excess of weight or an excess of buoyant force can occur at each vessel’s section along its length. Excessive load concentration at the front and rear ends of the vessel creates a hogging deformation. Meanwhile, excessive load concentration in the middle of the vessel creates a sagging deformation. In the long run, the uneven distribution of cargo weight across a chemical tanker’s compartments may result in the cracking of the vessel’s structure \((\text{Nugroho et al., 2018})\).

An inventory ship routing problem generally is experienced in industrial shipping when an owner is responsible for both managing inventory and transporting cargoes. This problem can be categorized as \textit{tactical} in maritime transportation planning \((\text{Christiansen et al., 2007})\). Meanwhile, a stowage planning problem can be classified as \textit{operational}, and operational problems are generally resolved after tactical problems have been solved. However, due to the aforementioned characteristics and key operational constraints of
chemical tankers, separating these two planning problems can lead to fleet routes for which no feasible stowage plan is possible.

The first model for a routing and scheduling problem facing a single chemical tanker, as well as a fleet of heterogeneous chemical tankers transporting multiple liquid chemical products, is proposed by Jetlund and Karimi (2004). Although they address the cargo routing problem for chemical tankers, they overlook chemical tankers’ uniqueness. Neo et al. (2006) formulate an extended version of the single chemical tanker cargo routing and scheduling model discussed in the previous paper by including additional constraints on cargo compatibility and vessel stability in their mixed-integer linear programming (MILP) model. This model is solved using commercial software considerably fast. A similar MILP model to the two in the previously mentioned papers is formulated by Cóccola and Méndez (2013), but it does not account for product compatibility and vessel stability. Oh & Karimi (2008) introduce what they call a “novel solution approach” to solve an industrial-scale chemical tanker routing and scheduling problem that accounts for product compatibility. They assume that vessel stability can be maintained within limits by filling a ballast tank adequately. All the above-mentioned routing and scheduling problems can be categorized as cargo routing problems (Al-Khayyal & Hwang, 2007).

Inventory ship routing and scheduling solutions for maritime chemical transport companies’ heterogeneous vessels transporting multiple liquid bulk products is proposed by Al-Khayyal and Hwang (2007) and Siswanto et al. (2011). Al-Khayyal and Hwang (2007) consider an MILP model to plan routes and schedules for multiple vessels carrying liquid bulk products in their multi-dedicated compartments, where each compartment is dedicated for a certain product. Siswanto et al. (2011) relax the previous problem by substituting multi-dedicated compartments with multi-undedicated compartments. They formulate the problem as an MILP and develop a multi-heuristics-based approach to solve it. Neither of these papers considers product compatibility and vessel stability constraints in their models.

Hvattum et al. (2009) introduce the problem of allocating bulk cargoes to compartments in a planned route maritime shipping, which is called the tank allocation problem (TAP). They consider product-compartment compatibility, compartment sloshing, stability, and hazmat regulation constraints in their MILP formulation and solve the problem using a commercial solver. Vilhelmsen et al. (2016) modify an optimality-based method presented in the previous paper. In contrast to the previous paper, they approach the TAP from a tactical viewpoint. Instead of identifying an optimal compartment allocation, their main objective is to swiftly determine feasible cargo allocations for a planned vessel route.

Decisions resulting from separate solution approaches to interrelated decision problems may not be compatible with each other. In this case, an integrated solution approach is needed that can solve problems simultaneously (Pasha et al., 2020). To our knowledge, no inventory ship routing problem has been formulated for chemical tankers that considers the stowage planning problem through integrated tactical planning. To bridge this research gap, we introduce a new mathematical model that integrates stowage planning, considering product compatibility, vessel stability, and durability as part of chemical tankers’ inventory routing when shipping multiple liquid chemicals.

The remainder of this paper is organized as follows. Section 2 describes the specific problem that we address. Our ISRPPP mathematical model is presented in Section 3. Section 4 discusses a case study. The results of our numerical experiments are presented and discussed in Section 5. Finally, Section 6 concludes this paper.
2. Problem Description

We describe the ISRSPP of a captive fleet consisting of heterogeneous types of chemical tankers simultaneously transporting different liquid chemical cargoes that cannot be mixed. These tankers are heterogeneous in size, load density, number and capacity of compartments, load capacity, cost, and time.

Their compartments are independent, which means they are not dedicated to specific cargoes. Each compartment can store a wide range of chemical cargoes, but only one at a time. Each compartment has its own load/discharge system with a dedicated pump and associated piping. We assume that all compartments are made of stainless steel. Incompatible cargoes must not be loaded into adjoining compartments.

Each port can be distinguished as either a producer or a consumer for specific products, and each such product has a certain daily production or consumption rate. Each product is stored separately into dedicated storage at port. The fleet travels from port to port to keep the inventory level of each product within its respective limits. Each tanker may fully or partially load one or more products at a producing port and then fully or partially unload one or more products at a consuming port. However, a tanker is permitted to serve a producing port if all of its compartments are empty. In the ballast condition, we assume that the vessel is trimmed by its stern to ensure a sufficient draft of the propeller.

Vessel stability requirements must be met every time a vessel serves a port call by properly distributing cargoes' weight. We assume that the first requirements for intact stability and damage stability have been complied with in the ship design process. In industrial shipping, a vessel is designed according to its owner's long-term needs to ship their cargo continuously. To reduce the free-surface effect, we assume that each compartment is subdivided longitudinally into several compartments of equal width by swash bulkheads that will hinder—but not prevent—the flow of liquid from side to side as the vessel rolls or heels. Meanwhile, cargo weight distribution must also consider maintaining vessel durability by preventing excessive load concentration either at both ends or in the middle of a vessel. We assumed no predetermined number of calls from each port, assignment of vessels to ports, or type and volume of cargoes to be handled at a certain port over the planning horizon.

Moreover, we assume that all incoming vessels can be served simultaneously. A vessel may delay its arrival at a consuming port until storage space is available to unload more cargoes. Likewise, it may also delay its arrival to a producing port to have an opportunity to load more cargoes. During these delays, we assume that a vessel will anchor at the recommended anchorage area. At the beginning of the planning horizon, vessels' initial location, initial quantity of any cargoes held in compartments, and initial inventory level for any products stored at the port are known. A vessel's initial location may be either a port or a point in the middle of the sea. Wherever it is located, we use a dummy port for each initial vessel location.

Our problem considers a captive fleet of vessels, as is common in inventory routing problems. Therefore, we assume that the dedicated fleet has sufficient capacity to maintain inventory levels at all ports over the planning period. Consequently, we omit the fixed costs of the fleet, including capital and running costs, that remain constant despite the fleet idling. We also exclude inventory carrying costs because the inventory at either producing or consuming ports belongs to the same company.

Based on the above-mentioned conditions, our problem is how to simultaneously determine (i) the tankers’ routes and delivery schedules so as to synchronize with the product inventory levels at all ports, (ii) the types and volumes of cargoes to be handled at producing or consuming ports, and (iii) the allocation of cargoes to compartments so as to
synchronize with chemical tankers’ unique characteristics and operational constraints. The integrated plan should minimize total voyage costs—which consist of traveling costs, anchoring costs, and port charges for the use of the port’s facilities and services—over a finite planning horizon. The port charges consist of port dues, which cover the cost of using the port infrastructure, and the cargo dues, which cover the cargo-handling costs.

3. Mathematical Model

Essentially, the inventory ship routing part of our ISRSPP mathematical model is in keeping with the work of Siswanto et al. (2011). All the notation and most of the constraints proposed in their paper remain valid here. However, we introduce some new notation due to chemical tankers’ characteristics and significant modifications to integrate the stowage planning part with the other aforementioned part of our problem, as shown below:

3.1. Notation

For ease of understanding, we first redefine all sets, indices, parameters, and variables introduced by Siswanto et al. (2011) to formulate additional or adjusted constraints in this paper.

Indices

\( i, j \) ports;
\( m, n \) port call numbers;
\( v \) vessels;
\( k \) products;
\( c \) compartments;
\( o(v) \) dummy start port of vessel \( v \).

Sets

\( N \) set of locations (port call number pairs);
\( N_p \) subset of locations at producing ports;
\( A_v \) subset of feasible links for vessel \( v \);
\( V \) set of vessels;
\( P_v \) subset of products that vessel \( v \) can carry;
\( K_i \) subset of products that port \( i \) can produce or consume;
\( C_v \) subset of compartments of vessel \( v \).

Parameters

\( CM_{vc} \) volume capacity of compartment \( c \) of vessel \( v \) (m\(^3\));
\( TQ_{ik} \) time required to load (or unload) one unit volume of product \( k \) at berth in port \( i \) (days);
\( TO_{ik} \) vessel time spent at berth in port \( i \) to prepare both the start of loading (or unloading) of product \( k \) and unberth of the vessel (days);
\( TT_{ijv} \) traveling time of vessel \( v \) between port \( i \) and port \( j \) (days);
\( TH \) planning horizon (days);
\( J_{ik} \) equals +1 (respectively, -1) if port \( i \) produce (respectively, consume) product \( k \), and 0 otherwise;
\( R_{ik} \) daily production (or consumption) rate of product \( k \) at port \( i \) (m\(^3\)/day);
\( SM_{ik} \) minimum inventory level of product \( k \) stored at port \( i \) (m\(^3\));
\( SX_{ik} \) maximum inventory level of product \( k \) stored at port \( i \) (m\(^3\));
\( CT_{ijv} \) traveling cost of vessel \( v \) from port \( i \) to port \( j \) (USD/day).

Variables

\( x_{imjn} \) equals 1 if vessel \( v \) sails from \((i, m)\) directly to \((j, n)\), and 0 otherwise;
\( z_{imv} \) equals 1 if node \((i, m)\) is the final location of vessel \( v \)'s route, and 0 otherwise;
$o_{imvk}$ equals 1 if vessel $v$ serves port call $(i,m)$ and product $k$ is loaded into (or unloaded from) its compartment $c$;
$q_{imvk}$ volume of product $k$ loaded into (or unloaded from) compartment $c$ of vessel $v$ at node $(i,m)$ ($m^3$);
$l_{imvk}$ volume of product $k$ held in compartment $c$ onboard vessel $v$ as that vessel leaves node $(i,m)$ ($m^3$);
$t_{im}$ arrival time of a vessel at node $(i,m)$ (hours);
$s_{imk}$ inventory level of product $k$ when a vessel arrives at node $(i,m)$ ($m^3$).

We define a new subset of all products that are incompatible with product $k$, namely $IP_k \subseteq K$. Figure 1 illustrates a chemical tanker and its dimensional parameters.

**Figure 1** Dimensional parameters of chemical tankers

We assume that all vessels and their compartments are box-shaped. A box-shaped vessel has a rectangular waterplane so that its center of flotation (COF) is situated on the centerline amidships. Depending on the strength of the compartment’s top, each compartment has a specific load density, which represents the cargo weight that can be safely loaded per unit volume of the compartment. Accordingly, new parameters follow:

- $L_v$ length between perpendiculars (LPP) of vessel $v$ (m);
- $B_v$ molded beam of vessel $v$ (m);
- $D_v$ molded depth of vessel $v$ (m);
- $MCTC_v$ moment to change trim 1 cm of vessel $v$ (tonnes m);
- $TR_0^v$ trim of vessel $v$ when all compartments are empty (cm);
- $TRB_{v}^{max}$ maximum absolute permissible trim by bow of vessel $v$ (cm);
- $TRS_{v}^{max}$ maximum absolute permissible trim by stern of vessel $v$ (cm);
- $LC_{vc}$ length of compartment $c$ of vessel $v$ (m);
- $BC_{vc}$ width of compartment $c$ of vessel $v$ (m);
- $DC_{vc}$ depth of compartment $c$ of vessel $v$ (m);
- $XF_{vc}$ longitudinal distance from aft perpendicular to front wall of compartment $c$ of vessel $v$ (m);
- $X_{vc}$ difference in longitudinal distance between compartment $c$ and vessel $v$ COF, where both are measured from aft perpendicular (m);
- $LD_{vc}$ load density of compartment $c$ of vessel $v$ (tonnes/m$^3$);
- $\rho_k$ specific gravity of product $k$ (tonnes/m$^3$);
- $\rho_{sw}$ specific gravity of sea water (tonnes/m$^3$);
- $CA_{iv}$ anchoring costs at anchorage area of port $i$ per day of vessel $v$ (USD/day);
- $CW_{iv}$ port $i$ dues of vessel $v$ (USD);
- $CO_{ik}$ cargo $k$ dues at port $i$ (USD).

We also introduce six new variables: $t_{im}^E$ and $t_{im}^A$ are related to scheduling constraints; $s_{imk}$ is related to inventory constraints; and the three other variables, $v_{im}$, $\tau_{im}$, and $d_{im}$, are related to modeling ship stability and durability constraints. The new variables are:
\( t_{im}^E \)

departure time of a vessel from node \((i, m)\) (hours);

\( t_{imv}^A \)

time spent at anchorage area by vessel \(v\) before arriving at \((i, m)\) (hours);

\( s_{imk}^E \)

inventory level of product \(k\) when a vessel leaves node \((i, m)\) (m³);

\( v_{imv} \)

equals 1 if vessel \(v\) has a negative change of trim when it leaves node \((i, m)\), and 0 otherwise;

\( \tau_{imv} \)

change of trim of vessel \(v\) when the vessel \(v\) leaves node \((i, m)\) (cm);

\( d_{imv} \)

change of draft of vessel \(v\) when the vessel \(v\) leaves node \((i, m)\) (m).

3.2. Constraints

The constraints in the mathematical model developed by Siswanto et al. (2011) are grouped into four parts: routing, loading and unloading, scheduling, and inventory. We adjust some constraints from their model and substitute them with new constraints to accommodate chemical tankers’ unique characteristics and operational limitations. We do not change at all the routing part of the work by Siswanto et al. (2011). However, we do significantly change three other parts. Furthermore, we introduce a new part of constraints related to the stowage planning problem.

3.2.1. Loading and Unloading Constraints

The volume capacity of compartment \(c\) of vessel \(v\), \(CM_{vc}\), can be calculated by multiplying \(LC_{vc}\), \(BC_{vc}\), and \(HC_{vc}\). Constraints 1 restrict that all compartments onboard must be empty when a vessel visits a production port. These constraints are non-linear, but they can be replaced with equivalent linear constraints, as illustrated by Al-Khayyal and Hwang (2007). Constraint sets 2 and 3 replace constraint sets 12 and 13 in the work of Siswanto et al. (2011). We use them to comply with the requirement regarding the load density of each vessel compartment. They ensure that the weights neither cargoes loaded (or unloaded) at node \((i, m)\) nor cargoes held onboard can exceed respective compartments’ capacities, which depend on these compartments’ respective load densities.

\[
\begin{align*}
z_{imv}(l_{imvkc}) &= 0, \forall v \in V, \forall (i, m) \in N - N_p, \forall (k, c) \in P_v \times C_v \\
\rho_k q_{imvkc} &\leq \sum_{(j,n) \in N \cup \{o(v)\}} LD_{vc}CM_{vc}x_{jnimv}, \forall v \in V, \forall (i, m) \in N, \forall (k, c) \in P_v \times C_v, i \neq j
\end{align*}
\]

(1)

(2)

3.2.2. Scheduling Constraints

We replace Constraints 24 in the paper by Siswanto et al. (2011) with constraints 4 and 5. Constraints 4 accommodate multiple cargoes’ simultaneous handling at berth while calculating the end of port call \((i, m)\). Meanwhile, Constraints 5 guarantee synchronization between routes and schedules. Constraints 5 are linearized the same way as Constraints 1. Constraint sets 6 and 7 restrict both new time variables within the planning horizon.

\[
\begin{align*}
t_{im} + TW_i \sum_{v \in V} w_{imv} + \max_{v \in V, k \in P_v} \left[TQ_{ik}q_{imvkc} + TQ_{ik}q_{imvkc}\right] &= t_{imv}^E, \forall (i, m) \in N \\
x_{imjv}(t_{im} + TT_{ijv} + t_{jnv}) &= 0, \forall v \in V, \forall (i, m, j, n) \in A, i \neq j \\
0 &\leq t_{im} \leq TH, \forall (i, m) \in N \cup \{o(v), 1\} \\
0 &\leq t_{imv} \leq TH, \forall v \in V, \forall (i, m) \in N
\end{align*}
\]

(4)

(5)

(6)

(7)

3.2.3. Inventory Constraints

We replace constraint sets 26, 28, 30, and 31 in the work of Siswanto et al. (2011) with constraint sets 8–12 due to \(t_{im}^E\) and \(s_{imk}^E\). Constraints 8 ensure that the loading quantity cannot exceed the available inventory at a producing port. Constraints 9 track product inventory levels at both the start and end of a port call \((i, m)\). Constraints 10 ensure the
consistency of inventory levels between successive port calls. Constraint Set 11 bounds the inventory level at the end of the planning horizon. Constraint Set 12 restricts the inventory levels at the end of a port call \((i, m)\) within respective limits.

\[
q_{imvkc} \leq s_{imk} + J_k R_{ik}(t^E_{im} - t_{im}), \forall v \in V, \forall (i, m) \in N_p, \forall k \in P_v, \forall c \in C_v
\]

\[
s_{imk} - \sum_{v \in V} \sum_{c \in C_v} J_k q_{imvkc} + J_k R_{ik}(t^E_{im} - t_{im}) - s^E_{imk} = 0, \forall (i, m, k) \in N \times K_i
\]

\[
s^E_{(m-1)k} + J_k R_{ik}(t_{im} - t^E_{im}) - s_{imk} = 0, \forall (i, m, k) \in N \times K_i, m \neq 1
\]

\[
SM_{ik} \leq s^E_{imk} + J_k R_{ik}(T - t^E_{im}) \leq SX_{ikv} \forall (i, m, k) \in N \times K_i, m = M_i
\]

\[
SM_{ik} \leq s^E_{imk} \leq SX_{ik}, \forall (i, m, k) \in N \times K_i
\]

### 3.2.4. Stowage Planning Constraints

The current subsection of this article describes additional constraints that we have developed to accommodate chemical tankers' operational limitations, including product compatibility, vessel stability, and durability. Constraints 13 obey the rules that incompatible chemical cargoes cannot be loaded into adjoining compartments. Equations 14 calculate the draft change due to the total weight of cargoes loaded onboard when vessel \(v\) serves a port call \((i, m)\). Equations 15 calculate changes to vessel trim caused by cargoes' weight distribution across compartments when vessel \(v\) serves a port call \((i, m)\) by dividing the trimming moment by the MCTC of vessel \(v\). To calculate the MCTC of a box-shaped vessel, we refer to the work of Barrass and Derrett (2012). In loading conditions, depending on cargoes' weight distribution across compartments, a change of trim may or may not occur, as depicted in Figure 2 and Figure 3. Constraint Set (16) restricts the change to vessel trim within respective limits in order to ensure stability.

Under loaded conditions, the heavier the cargoes onboard, the lower the vessel's position in the water and the greater the weight of the displaced seawater. However, the cargo allocation across compartments affects the shape of the immersed part of the hull, as Figure 2 and Figure 3 (the orange shaded parts illustrate). The distribution of the buoyant force acting on the underwater section is non-uniform but fixed because each section of the vessel experiences a downward force equal to the weight of water displaced by a transverse section of the corresponding section. We introduce constraint sets 17 and 18 to impose an even weight distribution of cargoes across compartments, such that the weight of cargo in each compartment must be at least equal to the buoyant force acting at respective underwater sections. Therefore, constraint sets 17 and 18 ensure that only a resultant downward force occurs in the form of excess of weight at each compartment section. We use the congruence rules of triangles to calculate the volume of water displaced in each compartment section, as Figure 4 shows. The uppercase \(M\) in constraint sets 17 and 18 denotes a very high positive number. Constraint sets 19–21 declare all variables involved in this part of the constraints.

\[
o_{imvk'c} + o_{imvk''(c+1)} \leq 1, \forall v \in V, \forall (i, m) \in N_p, \forall k' \in P_v, \forall k'' \in I P_k \cap P_v, \forall c \in C_v, k' \\

\neq k''
\]

\[
d_{imv} = \frac{\sum_{k \in P_v} \sum_{c \in C_v} \rho_k l_{imvk}}{L_v B_v \rho_{imv}}, \forall v \in V, \forall (i, m) \in N
\]

\[
\tau_{imv} = \frac{\sum_{k \in P_v} \sum_{c \in C_v} \rho_k l_{imvk} X_{vc}}{MCT_v}, \forall v \in V, \forall (i, m) \in N
\]

\[
(-TR_{vp}^\max - TR_{vp}^\min) v_{imv} \leq \tau_{imv} \leq (TR_{vp}^\max - TR_{vp}^\min) (1 - v_{imv}), \forall v \in V, \forall (i, m) \in N
\]

\[
\sum_{k \in P_v} \rho_k l_{imvk} = \left[LC_{vc} B_{vc} d_{imv} - \frac{1}{200} L_{vc} B_{vc} \tau_{imv} - \frac{LC_{vc} B_{vc} \tau_{imv}}{100 L_v} F_{vc}
\right.

\[\left. - \frac{LC_{vc}^2}{200 L_v} B_{vc} \tau_{imv} X_{vc}\right], \forall v \in V, \forall (i, m) \in N, \forall c \in C_v
\]
\[ \sum_{k \in P_v} \rho_{k} l_{imv_k} + \left[ LC_{vc} BC_{vc} d_{imv} + \frac{1}{200} LC_{vc} BC_{vc} \tau_{imv} - \frac{LC_{vc}}{100L_v} BC_{vc} \tau_{imv} XF_{vc} \right] \]

\begin{align*}
\rho_{sw} & \leq M v_{imv}, \forall v \in V, \forall (i, m) \in N, v_c \in C_v \\
v_{imv} & \in \{0, 1\}, \forall v \in V, \forall (i, m) \in N \\
\tau_{imv} & \text{ unrestricted, } \forall v \in V, \forall (i, m) \in N \\
d_{imv} & \geq 0, \forall v \in V, \forall (i, m) \in N
\end{align*}

(18)

(19)

(20)

(21)

**Figure 2** A box-shaped vessel under loading conditions with no change of trim

**Figure 3** A box-shaped vessel under loading conditions with a positive change of trim

**Figure 4** The shape of the immersed part of the box-shaped hull with a positive change of trim

### 3.3. Objective Function

We add anchoring an costs component, \( \Sigma_{v \in V} \Sigma_{(i,m) \in N} CA_{iv} \cdot \tau_{imv} \), to the objective function, as described by Siswanto et al. (2011). We also introduce new definitions of \( CW_{ip} \) and \( CO_{ik} \) to account for port \( i \) dues of vessel \( v \) and cargo \( k \) dues at port \( i \), respectively. Our objective function (22) minimizes total voyage costs during the planning period. The first term of the objective function corresponds to traveling costs, the second to anchoring costs, the third to port dues, and the final to cargo dues. We assume cargo dues are the fixed cargo-handling costs of loading (or unloading) product \( k \) at port \( i \).
\[
\min \sum_{v \in V} \sum_{i \in I, j \in J, m \in M} CT_{ijv}x_{imjn} + \sum_{v \in V} \sum_{i \in I, m \in M} (CA_{imv}t_{imv}^A + CW_{imv}w_{imv}) \\
+ \sum_{v \in V} \sum_{i \in I, m \in M} \sum_{k \in K, c \in C} CO_{ikc}\alpha_{imvkc}
\]

(22)

4. Case Study

We illustrate a small-case study of two heterogeneous chemical tankers (V1 and V2) carrying three liquid chemicals (K1, K2, and K3) between three ports (H1, H2, and H3). All products are stored in respective storage at all ports. H1 is a producing port, while H2 and H3 are consuming ports. Initially, V1 is at H1, and V2 is at H3. Table 1 shows the vessels’ detailed information.

**Table 1** Data for tankers V1 and V2

<table>
<thead>
<tr>
<th></th>
<th>V1</th>
<th>V2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars (m)</td>
<td>125</td>
<td>120</td>
</tr>
<tr>
<td>Molded beam (m)</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>Number of compartments (units)</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Moment to change trim 1 cm (tonnes m)</td>
<td>266.93</td>
<td>221.40</td>
</tr>
<tr>
<td>Trim in ballast condition (cm)</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Maximum absolute permitted trim by the bow (cm)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum absolute permitted trim by the stern (cm)</td>
<td>125</td>
<td>125</td>
</tr>
</tbody>
</table>

Table 2 presents detailed information about both tankers’ compartments.

**Table 2** Data for each compartment in tankers V1 and V2

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal distance from aft perpendicular to front wall (m)</td>
<td>27</td>
<td>58</td>
<td>79</td>
<td>24</td>
<td>50</td>
<td>81</td>
</tr>
<tr>
<td>Length of compartment (m)</td>
<td>30</td>
<td>20</td>
<td>30</td>
<td>25</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>Width of compartment (m)</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Depth of compartment (m)</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Longitudinal distance from COF to COG of compartment (m)</td>
<td>20.5</td>
<td>-5.5</td>
<td>-31.5</td>
<td>23.5</td>
<td>-5.0</td>
<td>-33.5</td>
</tr>
<tr>
<td>Volume capacity (m$^3$)</td>
<td>3,600</td>
<td>2,400</td>
<td>3,600</td>
<td>2,800</td>
<td>3,360</td>
<td>2,800</td>
</tr>
<tr>
<td>Load density of compartment (tonnes/m$^3$)</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>Initial level of cargo held in compartment (m$^3$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The specific gravity of Product 1 (K1), Product 2 (K2), and Product 3 (K3) are 1.25 tonnes/m$^3$, 0.85 tonnes/m$^3$, and 1.10 tonnes/m$^3$, respectively. K1 and K2 are incompatible chemical products. The maximum capacity, initial level, and daily production/consumption rate for each dedicated product storage at each port are presented in Table 3.

**Table 3** Data for each storage at ports H1, H2, and H3

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum capacity (m$^3$)</td>
<td>15,000</td>
<td>15,000</td>
<td>15,000</td>
<td>7,500</td>
<td>7,500</td>
<td>7,500</td>
<td>7,500</td>
<td>7,500</td>
<td>7,500</td>
</tr>
<tr>
<td>Initial level (m$^3$)</td>
<td>6,750</td>
<td>6,000</td>
<td>6,000</td>
<td>3,000</td>
<td>900</td>
<td>3,600</td>
<td>1,575</td>
<td>2,250</td>
<td>2,700</td>
</tr>
<tr>
<td>Daily production/consumption rate (m$^3$)</td>
<td>825</td>
<td>750</td>
<td>900</td>
<td>600</td>
<td>450</td>
<td>450</td>
<td>225</td>
<td>450</td>
<td>450</td>
</tr>
</tbody>
</table>

The traveling time between ports for both vessels is assumed to be one day. One day’s travel costs $10 for V1 and $8 for V2. The average total vessel time in port, minus the time at berth and the port dues, is one day and $1 at every port for V1, versus one day and $0.75
Inventory Ship Routing and Cargo Stowage Planning on Chemical Tankers

at every port for V2. The cargo dues for any products at any port are $0.50. The anchoring costs per day at each port anchorage area are $1 for V1 and $0.75 for V2. For simplification, we assume no time spent at berth to prepare either the start of loading (or unloading) and the vessel’s departure after finishing that activity and the time required to load (or unload) per unit volume of any cargoes at any port. We expect to determine the optimal inventory ship routes and schedules for a 15-day planning horizon.

5. Results and Discussion

We solve our model for two scenarios. In Scenario 1, we run our ISRSPP model as a whole, but we ignore the stowage planning part in Scenario 2. The MILP model for Scenario 1 has 4,038 constraints and 816 variables, including 387 binary variables. We used an AMD Ryzen 7 3700U 2,30 GHz processor to run LINGO 18 using the default option of the solver, and we obtained the global optimal solution in 11 minutes and 23 seconds. The total voyage costs are $73.00, which comprises $56.00 for traveling costs, $0 for anchoring costs, $6.50 for port dues, and $10.50 for cargo dues.

Figure 5 Vessels’ routes and schedules in Scenario 1

Figure 5 presents the optimal routing and scheduling solution for vessels V1 and V2 in Scenario 1. V1 remains at its initial position (H1), denoted by “(1,1),” which means that it is the first to arrive at Port H1. After loading cargoes, V1 travels to Port H2, denoted by “(2,1).” Then, it returns empty to port H1, denoted by “(1,3).” After loading the cargoes, V1 travels to Port 3, denoted by “(3,2),” and partially unloads its cargoes there. Then, it travels to Port H2, denoted by “(2,2),” and finishes its route there. V2 travels from its initial position (H3) to H1, denoted by “(1,2).” After loading its cargoes, V1 travels to Port H3, denoted by “(3,1),” and finishes its route there. V1 makes four journeys, while V2 makes only two journeys. Figure 6 shows that the inventory levels at all ports change over time throughout the planning period.

Figure 6 Inventory levels of all storage at all ports over the planning horizon

Figure 7 displays the inventory level of each compartment of both vessels. K1 and K2 cannot be loaded into neighboring compartments because they are incompatible. From Figure 7, we observe that cargo weight has been evenly distributed across both vessels’ compartments. We do not observe excessive weight for cargoes loaded at either end of the vessel and insufficient weight for cargo loaded amidship, and otherwise.
Meanwhile, in Scenario 2, the MILP model has 3,874 constraints and 816 variables, including 369 binary variables. We used the same processor to run the same software as in Scenario 1, and we obtained the global optimal solution in six minutes and 44 seconds. The total voyage costs are $65.25, comprising $50.00 for traveling costs, $0 for anchoring costs, $5.75 for port dues, and $9.50 for cargo dues. Table 4 shows the solutions for both scenarios.

Table 4 Key aspects of the solutions for the two scenarios

<table>
<thead>
<tr>
<th>Leg</th>
<th>Scenario 1</th>
<th></th>
<th>Scenario 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vessel (Node)</td>
<td>Loaded Cargo</td>
<td>Trim (cm)</td>
<td>U/E*</td>
<td>Vessel (Node)</td>
</tr>
<tr>
<td>1</td>
<td>V1(1,1)</td>
<td>K1 K3 K2</td>
<td>25.03</td>
<td>E</td>
<td>V1(1,1)a</td>
</tr>
<tr>
<td>2</td>
<td>V1(2,1)</td>
<td>K2 K3 K1</td>
<td>25.01</td>
<td>E</td>
<td>V2(1,2)a</td>
</tr>
<tr>
<td>3</td>
<td>V1(1,3)</td>
<td>nil</td>
<td>25.00</td>
<td>E</td>
<td>V1(2,1)</td>
</tr>
<tr>
<td>4</td>
<td>V1(3,2)</td>
<td>nil</td>
<td>25.00</td>
<td>E</td>
<td>V2(3,1)</td>
</tr>
<tr>
<td>5</td>
<td>V1(2,2)</td>
<td>nil</td>
<td>25.00</td>
<td>E</td>
<td>V2(3,1)</td>
</tr>
</tbody>
</table>

*U* indicates uneven weight distribution; *E* indicates even weight distribution.
aProduct compatibility constraints are violated.
bVessel stability constraints are violated.
cVessel durability constraints are violated.

Obviously, disregarding the stowage planning in Scenario 2 leads to an inventory routing and scheduling plan with a lower total cost than Scenario 1. However, the Scenario 2 plan clearly includes some loading (or unloading) activities that are not feasible because the corresponding cargo allocation across compartments violates product compatibility, vessel stability, or durability constraints. This finding illustrates the importance of blending chemical tankers’ stowage planning, inventory routing, and scheduling needs into one integrated planning, as our ISRSPP model shows.

6. Conclusions

In this paper, we developed a novel mathematical model ISRSPP that integrates our stowage planning problem and accommodates cargo compatibility, ship stability, and ship durability into the inventory routing and scheduling problem facing heterogeneous chemical tankers transporting multiple liquid chemicals. Integrating stowage planning...
from a tactical perspective, this model aims to suggest feasible vessel routes and schedules, rather than an optimal stowage plan. This model minimizes total voyage costs—including traveling costs, anchoring costs, port dues, and cargo dues—while satisfying constraints for routing, loading and unloading, scheduling, inventory, and stowage planning during the planning horizon. To the best of our knowledge, the literature has not presented a mathematical model that integrates inventory ship routing and stowage planning for chemical tankers.

We applied this MILP model to a small-case study solved in two scenarios using the LINGO 18 solver. Our results imply that cargo stowage planning cannot be separated from inventory ship routing and scheduling, particularly for chemical tankers, because such exclusion could create fleet routes for which no feasible stowage plan is possible. However, our model faces limitations, mainly due to considerable computation time, as our case study has shown. Obviously, this limitation must be addressed by developing specialized algorithms to exploit this model’s inherent structure before the model can be further developed so that it can solve more decision problems, such as ballast allocation, speed selection, fuel consumption, and weather routing. Such development would further expand the maritime transport sector’s resilience and sustainability.

Acknowledgements

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Marine Safety Committee, 2008b. 281 (85) Explanatory Notes to the SOLAS Chapter II-1 Subdivision and Damage Stability Regulation. *International Maritime Organization (IMO)*


