



## Achieving Drag Reduction with Hullform Improvement in Different Optimizing Methods

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**Abstract.** In general, evaluation of ship hydrodynamic efficiency could be produced by an energy-efficient and concentrated cost function. An optimization method with the representation of hull geometry is one of the preliminary design steps that are most appropriate for evaluating hydrodynamic performance. This work presents a comparison of two numerical methods for optimizing the shape of the hull concerning the minimization of total ship resistance in calm water conditions. The optimization method uses a theoretical approach based on Michell's integral and Rankine source methods. The discussion of the two methods emphasizes the comparison of wave resistance, total resistance, wave profiles, and wave contour. The optimized hull form comparison of total resistance between Michell's integral and Rankine source methods decreased by 3.79% and 4.0%, respectively. Comparing wave resistance with decreases by 5.52% based on Michell's integral method and 13.33% by the Rankine source method, the wave profiles generated by these two methods present a fair amount of compatibility. The wave contour illustrates a reasonably straightforward agreement on the optimal hull but are dissimilar on the initial hull.

**Keywords:** Michell's integral; Optimization; Rankine source method; Resistance; Series 60; Wave contour; Wave profiles

### 1. Introduction

The fundamentals of ship hydrodynamics are to obtain a design with minimum resistance following a specified speed and displacement. Total resistance is of the utmost importance for the ship, directly affecting speed, power requirements, and fuel consumption. The hydrodynamic performance of the ship can be enhanced by reducing friction and pressure resistance. Several recent techniques have been carried out to achieve reduced drag on ships, i.e., improvements to the hull structure (Ibrahim et al., 2018), micro bubble injection (Sindagi et al., 2018; Zhang et al., 2019), and optimization techniques (Park et al., 2015; Samuel et al., 2015; Choi 2015; Lu et al., 2016; Lu et al., 2019). The hull's geometric optimization is considered a relatively reliable and appropriate method of evaluating ship hydrodynamics. Objective functions, design variables, and limits to obtain optimal hydrodynamic efficiency concerning drag components and vessel performance, such as stability and seakeeping, are considered primary objective functions. It has supported computational optimization that has developed into a practical and fast design technique that automatically generates an optimal hull design to reduce drag. Fast -

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repetitive processes and reduced cost functions are the designer's choice for using this technique.

The advancement of Computational Fluid Dynamics (CFD) has expanded the domain of hydrodynamic problems effectively in viscous flow solving, domain decomposition, turbulence solver, and physical details of the phenomenon's flow field. The development of CFD computing technology has proven to be useful for evaluating the hydrodynamic performance of ships to produce an optimum hull and attempts to obtain a drag reduction (Yanuar et al., 2017; Wang and Yao, 2018; Zhang et al., 2019; Yanuar et al., 2020). The Rankine source method and Reynolds Averaged Navier Stokes (RANS) based viscous flow methods are potential flow panel methods that were developed in several studies with quite advanced techniques. The Rankine source method is considered fast, efficient, and highly precise in potential flow theory, e.g., Rankine source method with the optimization algorithm Nonlinear Programming Method (NLP) in monohull (Zhang and Zhang, 2015) and multihull optimization (Von Graefe et al., 2013; Von Graefe et al., 2015). The Michell integral method or thin ship theory is considered a more straightforward and faster CFD method (Tuck and Lazauskas, 1998). Several studies (Yanuar and Sulistyawati, 2018; Sulistyawati et al., 2020a; Sulistyawati et al., 2020b; Sulistyawati et al., 2020c) used Michell theory to investigate the hydrodynamic characteristics of pentamarans and compared them with experiments. Any deviations from the Michell integral method were deemed necessary for development. A boundary layer correction for potential flow or the tangency correction of the wave resistance oscillation problem at a small Froude number,  $Fr$ , in Michell's theory was delivered by (Bašić et al., 2018). However, these numerical results still require verification with experimental studies to test their validity.

This study represents a method of ship hull form optimization with the Michell integral. The hull is defined by inputting data with a grid offset setting into 21 stations and 21 water lines, a genetic algorithm in multi-objective optimizations to approximate the optimized hull with a minimum wave and total resistance in calm water. Two simple tools based on Michell's theory were quite applicable for investigating resistance performance and optimization (Sulistyawati et al., 2020b; Sulistyawati et al., 2020c). The results were compared with the Rankine source method (Zhang and Zhang, 2015). The Godzilla optimization tool (Lazauskas, 1996) and Flotilla (Lazauskas, 1999) were used for the optimization of the resistance components and contour of the wave elevation.

## 2. Methods

In nonlinear waves, Michell approached the boundary conditions on the hull's central plane ( $y = 0$ ), and the waterline ( $z = 0$ ) used the integral Fourier theorem to represent the linearized hull. Michell's theory for wave resistance ( $R_w$ ) at a specific speed with the parametric hull form ( $y = \pm Y(x, y)$ ) was expressed by:

$$R_w = \frac{2}{\pi} \rho U^2 k_0^4 \int_{-\pi/2}^{\pi/2} \sec^5 \theta d\theta \left| \iint_R Y(x, z) e^{ik_0 x \sec \theta + k_0 z \sec^2 \theta} dx dz \right|^2 \quad (1)$$

$Y(x, z)$  is data from offset ships with  $x$ , the length from the bow to the stern,  $y$  the length from the center to the right, and  $z$  up from the free surface;  $R$  the center position of the hull ship,  $U$  is the ship speed, and  $\rho$  is the water density. The Godzilla optimization tool uses a non-traditional Genetic Algorithm similar to (Scragg and Nelson, 1993) augmented with other features. The best design-vector for selecting a binary tournament with Stochastic Bit-climbing generates a candidate vector by varying each parameter's values. The

optimization minimizes the function of  $f(x_1, x_2, \dots, x_n)$ , with each parameter  $x$  subject to constraints  $a_i \leq x_i \leq b_i$ , where  $a_i$  and  $b_i$  are a value at bow and stern, respectively.

The center of the formation of a wave profile as  $z = \zeta(x, y)$  moves at various angles ( $\theta$ ) propagated relative to the negative  $x$ -axis to the ship's movement expressed by:

$$\zeta(x, y) = R_w \int_{-\pi/2}^{\pi/2} A(\theta) e^{-ik(\theta)\omega(x, y, \theta)} d\theta \quad (2)$$

where  $\zeta(x, y)$  represents the wave height or wave pattern at point  $(x, y)$ ,  $A(\theta)$  is the amplitude function with exponent,  $k_\theta = g/U^2$  is the transverse wavenumber ( $i$ ) at traveling angle ( $\theta$ ), and  $\omega$  as a phase function ( $\omega(x, y, \theta) = x \cos \theta + y \sin \theta$ ). Then, Equation 2 can be expressed as:

$$\zeta(x, y) = R_w \int_{-\pi/2}^{\pi/2} A(\theta) e^{-ik(\theta)[x \cos \theta + y \sin \theta]} d\theta \quad (3)$$

The optimization's objective function is total resistance ( $R_T$ ) so that the sum of wave resistance ( $R_W$ ) and viscous resistance  $R_V = R_F \cdot (k+1)$ . Where friction resistance ( $R_F$ ), Form factor ( $k+1$ ) is determined following (Eshelman and Schaffer, 1991),  $k = 0.0097(\theta_{entry} + \theta_{exit})$ , where  $\theta_{entry}$  and  $\theta_{exit}$  are the degrees of half-angles from the bow and stern at the waterplane. Thus, the total resistance coefficient  $C_T$  and  $C_w$  based on the thin ship is calculated by:

$$C_T = \frac{R_T}{0.5 \rho U^2 S} \quad (4)$$

$$C_w = \frac{R_w}{0.5 \rho U^2 S} \quad (5)$$

The ship wetted area is  $S$ , and wave resistance coefficient is  $C_w$ . In the Rankine source method, the factor  $k$  is calculated by:

$$k = \left( \frac{\nabla^{1/3}}{L} \right) \cdot \left( 0.5 C_b + \frac{2\gamma^{1/3}}{C_b} \right) \quad (6)$$

$$\gamma = \frac{(b/L)}{(1.3(1-C_b) - 0.031 l_{cb})} \quad (7)$$

where

volume displacement is  $\nabla$ , hull breadth is  $b$ , the block coefficient is  $C_b$ , the degree of the stern is  $\gamma$ , ship length is between perpendicular  $L$ , and the longitudinal position of the center of buoyancy is  $l_{cb}$ . The formulation of friction resistance coefficient ( $C_F$ ) for both Michell integral and Rankine source methods uses a standard of the 1957 ITTC that is:

$$C_F = 0.075 / (\log_{10} \text{Re} - 2)^2 \quad (8)$$

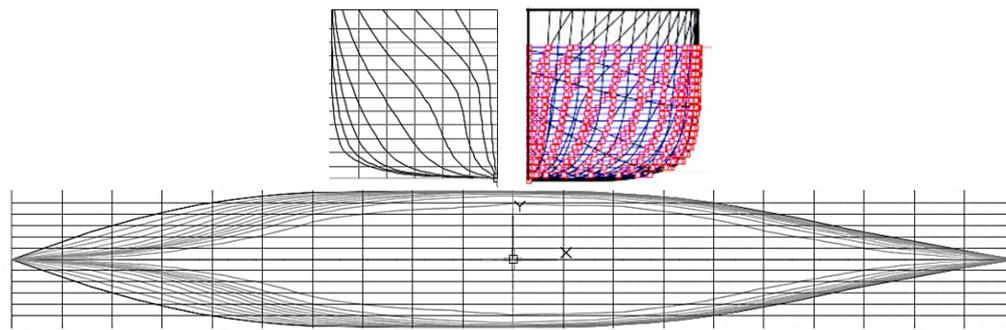
### 3. Comparison of Two Optimization Methods

The hull used series 60 with a total length of 2 m, breadth of 0.267 m, depth of 0.137, and draft of 0.107 (Table 1). Figure 1 shows the first model lines plan as an initial/ parent hull. The Godzilla optimizations used a speed range of 1.0–1.7 m/s, and corresponding to  $Fr$  0.22–0.38. The optimization process took several minutes to evaluate hundreds of iterations, with the population highlighted in green, indicating that the optimization

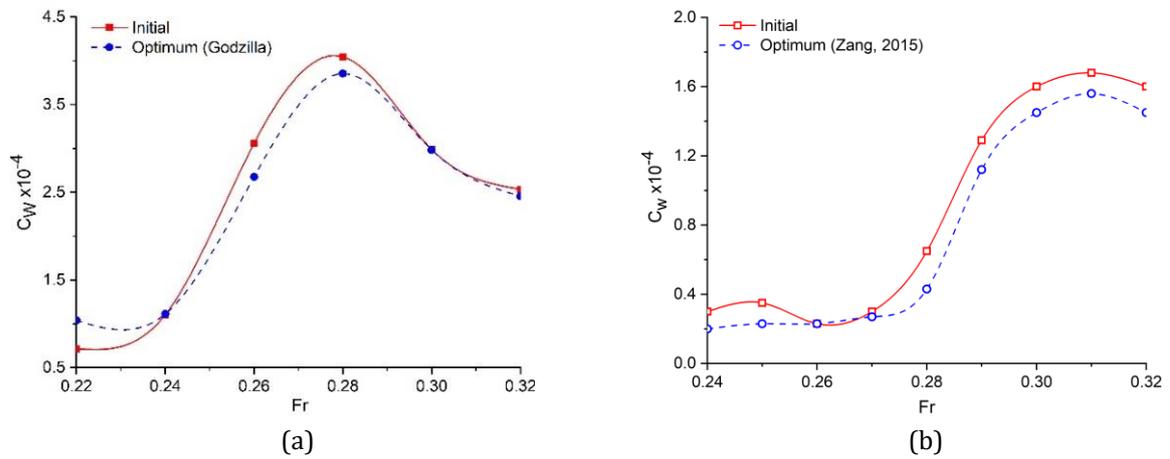
parameter provided a convergent value. The convergence of optimization verifies the minimum wave resistance, the waterline's repetition process, and the section of the hull for varying numbers 33 to 81. The components of resistance coefficient calculation by comparing Michell's integral and Rankine source methods are presented in Figures 3–6.

**Table 1** Principal dimensions of hull Series 60

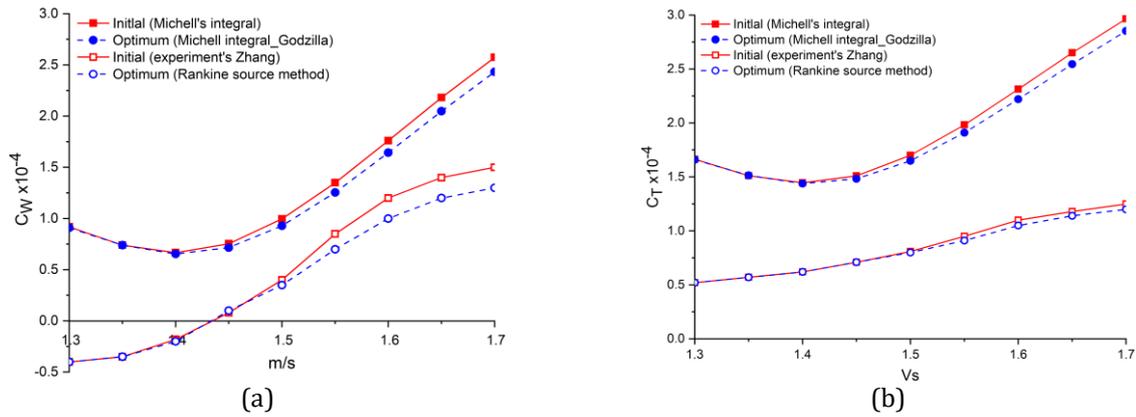
Dimension	Symbol	Value (m)
Length overall	<i>Loa</i>	2
Breadth	<i>B</i>	0.267
Depth	<i>D</i>	0.137
Draught	<i>T</i>	0.107
Coefficient block	<i>Cb</i>	0.6



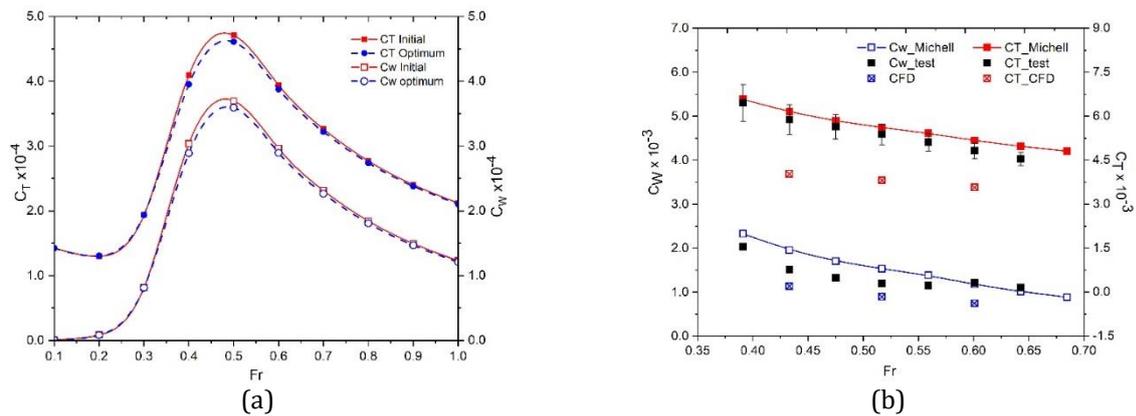
**Figure 1** Lines plan of hull Series 60 with offset set of hull station and waterline



**Figure 2** Wave resistance coefficient ( $C_w$ ) based on: (a) Michell integral; and (b) Rankine source method



**Figure 3** Comparison of Michell's integral method and study of (Zhang and Zhang, 2015): (a) wave resistance,  $C_w$ ; and (b) total resistance,  $C_T$



**Figure 4** Comparison of: (a) wave resistance,  $C_w$ , and total resistance,  $C_T$ , of hull series 60; and (b) optimization with the Michell integral method, test results, and RANS-CFD (Sulistyawati et al., 2020c)

Comparison of the wave resistance coefficient based on Michell's integral is presented Figure 2a, while Rankine source methods in Figure 2b. The wave resistance coefficient ( $C_w$ ) and total resistance coefficient ( $C_T$ ) are compared to that in Zhang's research (Zhang and Zhang, 2015) in Figure 3. Figure 2 shows that the wave resistance coefficient ( $C_w$ ) in the Michell integral method is higher than that in the Rankine source method and provides an entirely different graph. Nevertheless, the optimization of Michell's integral method displays decreased wave resistance between the initial and optimum hull. This method is not appropriate for very low speeds or at  $Fr < 0.4$ , while in Zhang's research, there was no analysis carried out at high speeds ( $Fr > 0.32$ ). The significant deviation of wave resistance from the Michell integral method at  $Fr$  2.85 decreased 4.71%, and the Rankine source method at  $Fr$  3.0 decreased by 9.3%. The significant difference in  $C_w$  ( $V_s$  1.7 m/s) in Figure 3a with the experiment on initial and optimum hull based on Michell was 49.5% and 46.5%, respectively. In comparison, the deviation of total resistance coefficient,  $C_T$ , with the experiment in Figure 3b on initial and optimum hull based on Michell was 59.5% and 57.9%, respectively. The total resistance of the optimized hull form between Michell's integral and Rankine source methods decreased by 3.79% and 4.0%, respectively. The wave resistance decreased by 5.52% based on the Michell integral method and 13.33% in the Rankine source method.

The investigation of hull series 60 at a higher speed than Zhang's research using the estimate of Flotilla based on Michell's thin ship (Figure 4a) for total resistance coefficient

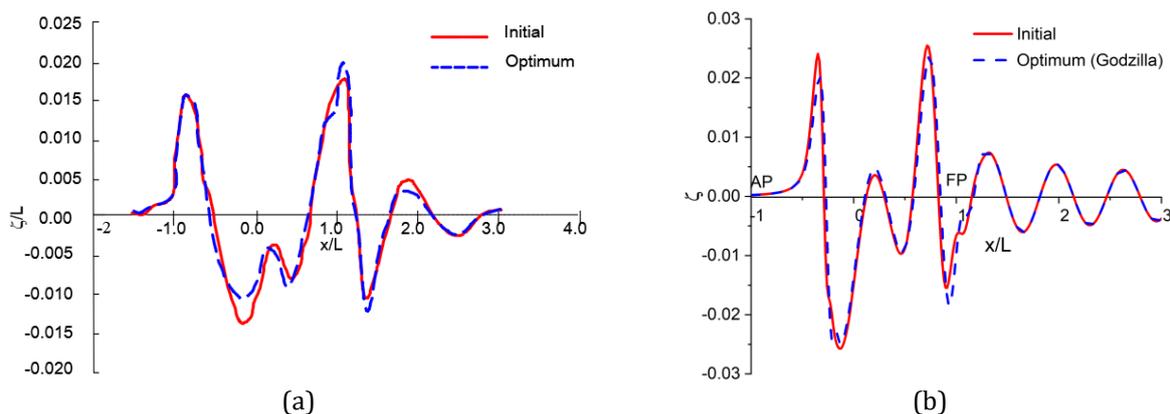
and the wave resistance coefficient at variations  $Fr$  0.1–1.0. Where generally, the near-thin hulls are used for high-speed ships. In increasing speed, the graphs coincided with each other with a low indication of difference. The deviation of the total resistance coefficient in Figure 4a at hump ( $Fr$  0.5) decreased by 2.1%. The same consequences were obtained in the wave resistance coefficient, which was reduced by 2.8%. Due to the COVID-19 pandemic, high-speed testing was not conducted. However, the first authors (Sulistyawati et al., 2020c) proved that the Michell integral method is quite precise with test results at higher speeds above  $Fr$  0.4. Analysis of pentamaran configuration were optimized with the Michell integral method and were closer to the test results than the Reynolds Averaged Navier Stokes equations by Computational Fluid Dynamics or called RANS-CFD (Figure 4b).

#### 4. Wave Profiles and Contour

The wave-cut analysis predicted wave resistance with a longitudinal or lateral cut of the wave at specific points  $x$  and  $y$  in the wave profile. Determination of the  $x$  and  $y$  positions is taken with the assumption that wave fluctuations occur at these points. The wave's near-field profiles were based on Michell's integral method taken by the longitudinal wave-cut method at position  $y/B$  0.5 from the centerline at  $Fr$  0.32. The capture of wave contour from Flotilla was taken at position  $x/L$ : -2 to 3 from the model at  $Fr$  0.3. The wave profile and contour by the Rankine source method of Zhang's research (Zhang and Zhang, 2015) and Michell's integral method are presented in Figure 5.

##### 4.1. Wave Profiles based on Rankine Source and Michell's Integral Methods

The results clearly show that the hump and hollow of the wave at each  $Fr$  value of the two methods occur at an almost identical position  $x/L$ , although the waves' height is unequal.



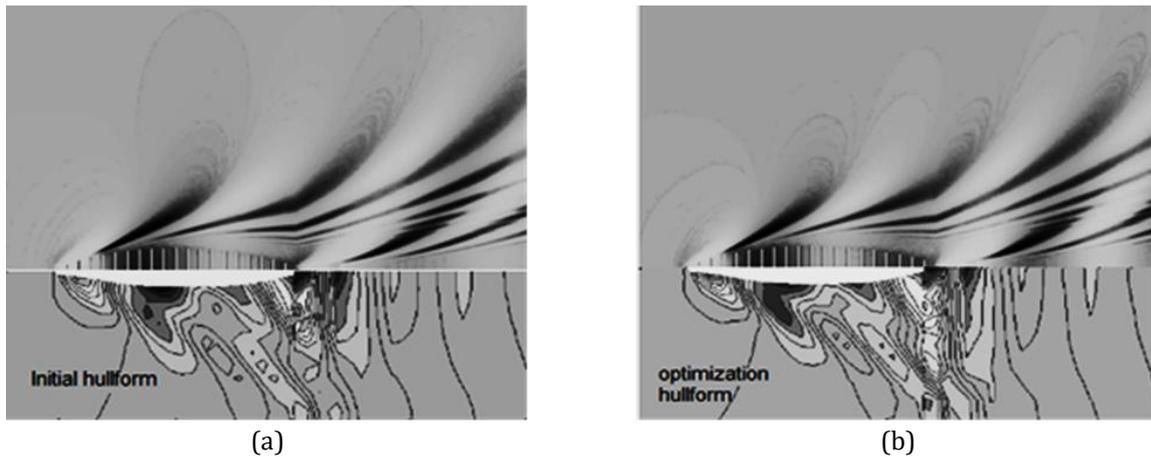
**Figure 5** Comparison of the wave profile based on: (a) Rankine source (Zhang and Zhang, 2015); and (b) Michell integral methods

The wave profile by the Rankine source method in Figure 5a shows a reasonably clear difference between the initial and optimum hull, in contrast to the results of the Michell integral method in Figure 5b, which does not indicate a difference in the wave profile. Even though the calculation gives a different value, the estimation of wave cutting shows the wave's profile line is quite similar to Rankine's method profile.

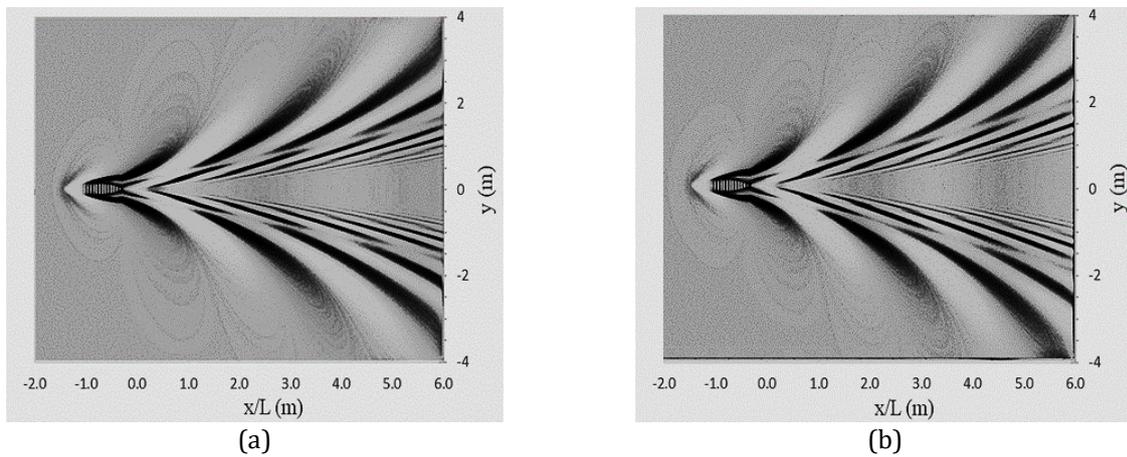
##### 4.2. Wave Contour based on Rankine Source and Michell's Integral Methods

The wave contour describes the wave resistance generated by the ship. The comparison of wave contour of hull series 60 using the Michell integral method from Flotilla and the Rankine source method is presented in Figure 6. A contour wave was

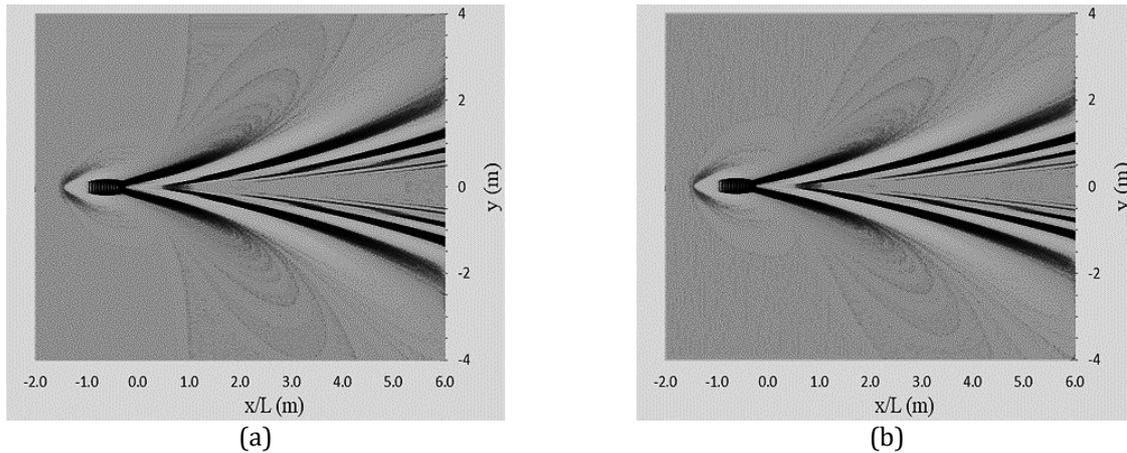
captured at position  $x / L$ : -0.5 to 1.5 from the hull at speed 1.7 m/s ( $\approx Fr$  0.38). The wave contour layer (Figure 6) on the sectoral patch area at the rear of the ship is a linear superposition of the near-field wave. Its contours displayed black-white colors, indicating black as the deepest trough and white as the wave's highest peak. The two methods' wave contour illustrates a reasonably straightforward agreement on the optimal hull, even though it looks different on the initial hull. The contour of the Michell integral method gives a clear distinction between the initial and optimal hulls. However, the Rankine source method seems to have no apparent difference in the contour of both hulls. Nevertheless, using an exact capture of the contours' transverse and divergent waves, this method seems suitable for numerically solving wave-body interaction problems at low speeds.



**Figure 6** Comparison of wave contour between Michell integral (top) and Rankine source methods (bottom) on: (a) initial hull; and (b) optimum hull



**Figure 7** Contour wave based on the Michell integral method at  $Fr$  0.6: (a) initial hull; and (b) optimum hull



**Figure 8** Contour wave based on the Michell integral method at  $Fr$  1.0: (a) initial hull; and (b) optimum hull

The wave contour was captured at a higher speed ( $Fr$  0.6 and 1.0) using the Michell integral method for the initial and optimum hull (Figures 7–8). The wave contour in Figure 7 shows conformity to the wave coefficient results shown in Figure 4a; the optimal hull produced waves smaller than the initial hull at  $Fr$  around 0.6. At  $Fr$  1.0 (Figure 8), both wave contours had a similar layer but still provided a difference from the optimal hull, producing a shorter/smaller wave compared to the initial hull.

## 5. Discussion

Optimizing the parametric hull  $y = \pm Y(x, y)$  minimized the function of  $f(x_1, x_2, \dots, x_n)$  to the complexity effect of amplitude on boundary layers and wave from the hull shape  $z = z(x, y)$ . Therefore, the hull's optimum shape was a decreasing representation of the boundary layer's complexity effect and shape; its benefit was achieved at a sufficiently high speed, where the viscous effect was minimal. Optimizing the hull based on the Michell integral method by investigating resistance performance compared with the experimental data of (Zhang and Zhang, 2015) revealed quite a deviation. It indicated that the Michell integral method was inadequate for analyzed resistance at very low speeds because it neglected the nonlinear viscous effects, while the viscosity factor dominated at low speeds. Investigating the wave profile showed that the wave profile from the Michell integral method had no reasonably clear difference between the initial and optimum hull, while there was a striking difference using the Rankine source method. It is likely that the wave-cut position did not accurately point to the wave-cut post from research of (Zhang and Zhang, 2015). Furthermore, capturing wave contours from the Michell integral method provided a more evident difference and described the differences according to the resistance analysis results. Conversely, Rankine's contour capture did not significantly differ between the initial and optimum hulls.

## 6. Conclusions

Conforming to this study's purpose, which investigated the comparison of Michell's integral theory and the Rankine source method, several analyses were carried out on the total resistance, wave resistance, wave profile, and its contours. The optimal model produced by the two methods showed good graphical conformity even with significant differences. Unfortunately, the research of (Zhang and Zhang, 2015) were not carried out at a higher speed,  $Fr > 0.32$ . In contrast, the approach with the Michell integral method was

deficient at low speeds. Theoretically, the Michell integral method linearizes the shape of the hull and free surface conditions. The form factor approach is perhaps less precise, and the friction factor dominates at low speed. It is, therefore, very likely that this is the reason for a considerable discrepancy between the two methods. The Rankine source method considers the nonlinear on the actual free surface and nonlinear hull surface conditions.

Improvements in the complicated numerical Michell integral should consider the tool's viscous and nonlinear effects, which is needed to obtain more accurate results. Computation between the optimization of these two methods showed differences in the resistance component, wave profile, and contour. It is necessary to review the subsequent analysis of water conditions and the towing experiment at a higher speed.

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