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NEW APPROACH TO DETERMINING MATHEMATICAL EQUATIONS FOR OPTIMUM TILT ANGLE OF SOLAR PANELS IN INDONESIA AND ITS ECONOMIC IMPACT

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ABSTRACT
The findings in this study refute mathematical equation from existing research that has been determining the optimum tilt angle of the solar panel with a subtropical location perspective. This research found that the influence of degrees latitude (Y) and longitude (X) to the optimum angle of solar panel installation in the territory of Indonesia (tropical location) can be represented by the equation 0.0093 X + 1.3042 Y, with an RMSE value of 1.88 and an R squared value of 0.928. However, mathematical equations for optimum tilt angle based on the coordinates of locations in 9 different prefectures of Indonesia resulted in better RMSE and R squared values. The maximum potential economic benefit to be gained from the installation of solar panels at the optimum angle in Indonesia, assuming a feed-in tariff in Indonesia of US$ 0,25 and a solar panel capacity of 1 MW and production life of 20 years, is US$ 740.839,66.

Keywords: Latitude; Longitude; Optimum tilt angle; Solar panel

1. INTRODUCTION

Solar energy has good potential for development in Indonesia for at least two reasons. First, Indonesia is located near the equator, from 6° North latitude to 11° South latitude and from 95° East longitude to 141° East longitude. This geographic location receives stable sunshine with good intensity throughout the year; the average value of solar radiation in Indonesia is 4.8 $kWh/m^2/day$. Second, Indonesia is the largest archipelago in the world. If viewed from a centralized network perspective, this characteristic would be an economic drag. However, when viewed from a distributed network perspective, renewable energy, particularly solar, is an opportunity (Perusahaan Listrik Negara, 2015–2024; Perusahaan Listrik Negara, 2013).

One way to encourage the use of solar energy is to provide data on solar radiation throughout Indonesia to industries that want to use this technology. However, solar radiation data are currently available only for horizontally and vertically positioned solar panels. In actuality, solar panels are installed at an optimum tilt angle. The coordinates of a place affect the amount of solar radiation received and the optimum angle of solar panel installation.

Geometrically, there are at least three things that affect this optimum angle: declination angle (the angle of the Earth as it rotates on its axis), latitude angle, and the angle of the surface of the radiation receiver.

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Most research has investigated the optimum tilt angle of solar panels in subtropical locations because most researchers live in subtropical countries. This is illustrated by the recommended magnitudes of solar panel tilt angles, which indicate the influence of locations in the high latitudes. These recommended slopes are not necessarily relevant to the tropics, nor in particular to Indonesia.

Table 1 Recommended optimum angles from various sources

Source	Recommended
Duffie et al. (1980)	$(\phi + 15°) \pm 15°$
Heywood (1971)	$(\phi\ !10°)$
Lunde (1980)	$(\phi \pm 15°)$
Chinnery (1971)	$(\phi + 10°)$
Lof & Tybout (1973)	$(\phi + 10° \rightarrow 30°)$
Garg (1982)	$(\phi \pm 15°)$

Note: ϕ is the latitude coordinate

Implementation of solar technology is usually reliant on economic factors, particularly the cost of energy needed to produce energy from solar panels (USD/kWh). This indicates the need to map areas of potential solar panel use from a cost-of-energy viewpoint. Angle adjustments create opportunities to reduce energy costs due to an increase in the value of solar radiation received.

2. EQUATION TO DETERMINE TILT ANGLE

Research on optimum tilt angles has used several approaches, including determining the optimum angle based on the season (usually summer and winter) or the month or attempting to determine an optimum fixed angle. The technical implementation of solar panels makes it difficult to change the angle each month or even every 6 months. Therefore, this study uses a fixed-angle approach.

Handoyo and Prabowo (2012) recommend an optimum angle for the city of Surabaya (7.2° LS) of 0°–40° for March 12th to September 30th and 0°–30° for October 1st to March 11th. Siraki and Pillay (2012) claim that the annual optimum tilt angle for lower latitudes is equal to the latitude of the location, while at higher latitudes the value is less than the value of the latitude. Khalifa (2011) states that research results tend to show that the value of the optimum tilt angle is close to the value of the latitude angle. For example, the optimal tilt angle at latitude 15° N is about 19°, while at 35°N latitude the optimal tilt is approximately 32°.

Stanciu and Stanciu (2014) use two different research methods and recommend two mathematical equations:

$$\beta = \phi\ !\ \delta \qquad (1)$$

if the result if positive, or

$$\phi + 10 \qquad (2)$$

The research results of Portolan dos Santos and Rüther (2014) in Brazil show that the optimum angle can be obtained by the equation:

$$\beta = \phi \quad (3)$$

and sometimes

$$\beta = \phi \; ! \; 10 \quad (4)$$

Results of other studies stating that the equation for the optimum angle is $\beta = \phi$ include Al Hinai et al. (2002), Singh and Tiwari (2004), and Aybar and Assefi (2009). In addition to Portolan dos Santos and Rüther (2014), another researcher who provided recommendations on the corner of the optimum use of case studies in subtropical locations. Darhmaoui and Lahjouji use a quadratic regression approach to determine mathematical equations for the Mediterranean region (2013). The purpose of the current study was to determine the equation for optimum tilt angle specifically for locations in Indonesia.

3. METHODOLOGY

A literature study was conducted to ascertain the effect of geographical location (coordinates) in Indonesia on the value of solar radiation received. This influence, which is expressed as a mathematical equation, assisted in conducting research iterations to find the optimum angle for solar panel installation. The equation was used to process NASA vertical radiation data. In addition, previous NASA data was validated by analyzing the error when compared with the measured data at certain points throughout Indonesia (Sanders & Joannde, 2015; NASA, 2014).

After processing the data, the optimum angle for solar panel installation was mapped for all parts of Indonesia from 6° North latitude to 11° South latitude and 95° East longitude to 141° East longitude (in 1° increments). To determine a general equation for the data pattern, the data was then classified into several prefectures, and the discretion error value was found by using a mathematical equation created with the Best Linear Unbiased Estimator (BLUE) method.

The results of optimum angle mapping and determination of the mathematical equation for the data pattern were followed by an analysis of the sensitivity of adjustments to the optimum angle of solar radiation received by the solar panels. To calculate economic aspects, we investigated technical data and economic use of solar panel systems (Solar Power Plant On-Grid without battery). The data were then used for mapping of areas that can be reached with the lowest Cost of Energy (COE).

In general, solar radiation is influenced by three things, namely beam radiation, global radiation, and diffuse radiation. Beam radiation is solar radiation that is received directly by the surface, diffuse radiation is solar radiation received from reflection, and global radiation is the total radiation received. Solar radiation takes into account three parameters associated with the three types of radiation. The link in question is the relative position or tilt of the solar panels in relation to the source of each type of radiation (Duffie et al., 1980).

- The ratio of the slope of the solar panels to the beam radiation (R_{bm})

$$R_{bm} = \frac{\omega stm \ \sin\delta m \ \sin(\phi - \beta) + \cos \ \delta m \ \sin \ \omega stm \ \cos(\phi - \beta)}{\omega sm \ \sin \ \delta m \ \sin\phi + \cos \ \delta m \ \sin \ \omega sm \ \cos \ \phi} \quad (5)$$

Where ϕ is the latitude of the location of the solar panels and ω_{stm} and ω_{sm} is the rising and setting of the sun. ω_{stm} and ω_{sm} are found with the following equations:

$$\omega_{stm} = \cos!1[!\tan(\phi \; ! \; \beta) \tan \delta_m] \quad (6)$$

$$\omega_{sm} = \cos!1[!\tan\phi \tan \delta_m] \quad (7)$$

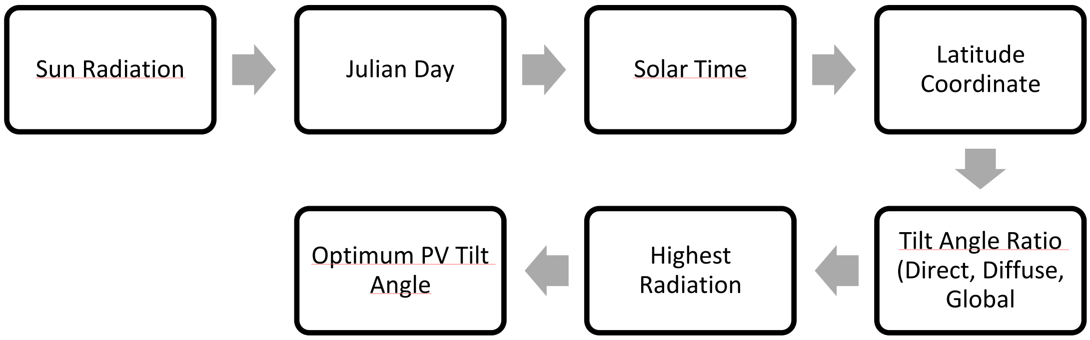

Figure 1 Calculation flowchart

- The ratio of the slope of the solar panels to the diffuse radiation (R_{dm})

$$R_d = \frac{1+\cos\beta}{2} \quad (8)$$

- The ratio of the slope of the solar panels to the global radiation (R_{rm})

$$R_r = \frac{\rho 1(1+\cos\beta)}{2} \quad (9)$$

where ρ1 is the reflection factor of the surface.

After finding the value of these variables, the total potential energy of solar radiation can be obtained by the following equation:

$$H_{Tm} = H_{bm}\, R_{bm} + H_{dm}\, Rd + H_{gm}\, R_r \quad (10)$$

By calculating the value of solar radiation for each solar panel slope, the optimum angle required to produce the most energy can be identified.

4. RESULTS AND ANALYSIS: GENERAL EQUATION FOR OPTIMUM SOLAR PANEL TILT ANGLE IN INDONESIA AND ITS EFFECTS

4.1. Mapping the Optimum Tilt Angle for Solar Panels in Indonesia

NASA data is secondary data that needs to be validated from direct measurements of solar radiation. Measurement data was obtained from the Agency for Assessment and Application of Technology and the Ministry of Energy and Mineral Resources for 17 locations throughout Indonesia. Root Mean Square Error (RMSE) was used to analyze the error value calculation. The RMSE analysis shows that the average calculation error for NASA data in comparison to measurement data at 17 locations in Indonesia is 0.73.

In general, the results show that the farther north an area is, the greater the number of solar panels that need to be installed tilted to the south. Conversely, the more southern the latitude of an area, the greater the number of solar panels that need to be installed tilted to the north.

The BLUE method was then used to determine the mathematical equation that best fit the pattern of data in Indonesia. This research is closer to the data pattern with a quadratic equation of two variables, or *z* in function of *x* and *y* where *x* is the overall degree of longitude and y is the degree of latitude.

Table 2 Error analysis between NASA data and Indonesian radiation measurement data

Province	Location	Measured radiation intensity	Radiation intensity of NASA data	Error	(Measurement NASA data)^2
NAD	Pidie	4.710618342	4.097	13%	0.376527469
SumSel	Ogan Komering Ulu	4.635833378	4.573	1%	0.003948033
Lampung	Kab. Lampung Selatan	4.715834162	5.234	11%	0.268495836
Banten	Lebak	5.078335365	4.446	12%	0.399848014
Jawa Barat	Bandung	5.174168464	4.149	20%	1.050970380
Jawa Tengah	Semarang	5.231668134	5.488	5%	0.065706026
DI Yogyakarta	Yogyakarta	5.479169213	4.500	18%	0.958772347
Jawa Timur	Pacitan	5.575002829	4.300	23%	1.625632214
KalBar	Pontianak	5.257500792	5.552	6%	0.086729784
KalTim	Kabupaten Berau	4.779166667	4.172	13%	0.368651361
KalSel	Kota Baru	4.922500543	4.573	7%	0.122150630
Gorontalo	Gorontalo	5.911820660	4.911	17%	1.001641993
SulTeng	Dinggala	5.109166667	5.512	8%	0.162274694
Papua	Jayapura	6.134171729	5.720	7%	0.171538221
Bali	Denpasar	6.170838283	5.263	15%	0.824170348
NTB	Kab. Sumbawa	5.631672655	5.747	2%	0.013300397
NTT	Ngada	6.445009167	5.117	21%	1.763608347
		RMSE			0.738199961

The data pattern for the optimum angle of installation of solar panels in Indonesia resulted in the following equation:

$$!0.0093 X + 1.3042 Y + 0.0002 \quad (11)$$

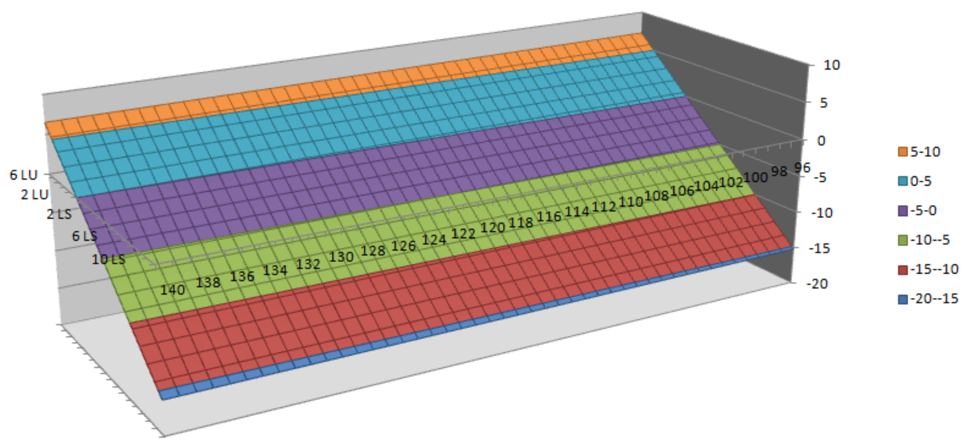

Figure 2 Indonesian optimum angle mapping, where the X axis is longitude (degrees) and the Y axis is latitude (degrees)

To ensure that the linear equations of two variables have a representative value, statistical error analysis is required. In this study, the measurement error value using RMSE was 1.88.

To perform correlation analysis, the *R* squared value was calculated, and an *R* squared value of 0.928 was determined. The optimum tilt angle for solar panel installation depends on the coordinates of the location (latitude and longitude) for 92.8% of its value and on other variables for 7.2% of its value. In addition to the coordinates of the location, other variables which might influence optimum tilt angle are weather characteristics in the vicinity (Solar Energy

International, 2013). However, the *R* squared value of 0.928 indicates that there is a close relationship between the optimum solar panel tilt angle and location coordinates. In other words, equations created by using location coordinate (latitude and longitude) variables have a strong correlation with the value of the optimum tilt angle.

To test the hypothesis that a variable (X or Y) does not affect the other variables (Z), assuming that the other variables are constant, the constant (B) was formulated for the following conditions:

H_0: B = 0 (no effect)
H_0: B > 0 (has positive effect)
H_0: B < 0 (has negative influence)

To test hypotheses about the regression coefficient, t-test statistics were used, with the T_0 value compared with t_α or $t_\alpha/2$, which was obtained from the t distribution table.

The N value used was 828, and k was 2, so that n ! k = 826. This value is large enough so that it can be assumed to be ∞.

- For b_2
  $\alpha = 0{,}05 \rightarrow !t_{0.05(\infty)} \rightarrow !1.645$
  t_0 pada b_2 = !4.38 < $!t_{0,05(\infty)} \rightarrow !1.645$

Thus, H_0 is rejected. Therefore it can be concluded that there is a negative effect of longitude coordinates on energy consumption (rejecting H_0 means that Hα is not refused).

- For b_3
  $\alpha = 0{,}05 \rightarrow t_{0.05(\infty)} \rightarrow 1.645$
  t_0 pada b_2 = 2.95 > $!t_{0,05(\infty)} \rightarrow 1.645$

Thus, H_0 is rejected. Therefore it can be concluded that there is a positive effect of longitude coordinates on energy consumption (rejecting H_0 means that Hα is not refused).

These findings refute the mathematical equations for determining the optimum angle for solar panel installation supplied by Duffie et al.(1980), Heywood (1971), Lunde (1980), Chinnery (1971), Lof and Tybout (1973), and Garg (1982), and suggest that the proposed mathematical equations to determine the optimum tilt angle are not relevant for Indonesia.

4.2. Indonesia Area Classification

For the purposes of this study, Indonesia was divided into several prefectures. The goal was to look for relevant patterns of data based on the distribution of data in longitude coordinates. Indonesia's vast extent means that the spread of the data is too wide to be encompassed by a single equation describing the relationship of longitude coordinates to the optimum angle. In this study, the desired standard deviation value was less than 2. The prefecture division was as follows:

Prefecture 1	95° BT	–	101° BT
Prefecture 2	101° BT	–	110° BT
Prefecture 3	110° BT	–	113° BT
Prefecture 4	113° BT	–	115° BT
Prefecture 5	115° BT	–	117° BT
Prefecture 6	117° BT	–	120° BT
Prefecture 7	120° BT	–	126° BT
Prefecture 8	126° BT	–	134° BT

Prefecture 9 134° BT – 141° BT

The results show better values of RMSE and *R* Squared for prefecture equations versus use of one equation for all of Indonesia (see Table 3).

Table 3 Comparison between one equation for Indonesia and prefecture equations

Methodology		Equation	RMSE	R Squared
One Equation		-0.0093 X + 1.3042 Y	1.88	0.928
Prefecture Equation	1	-0.000103 X + 0.11897 Y	1.22	0.958
	2	-0.00023 X + 0.12286 Y	1.21	0.956
	3	-0.000229 X + 0.12956 Y	1.71	0.967
	4	-0.000177 X + 0.13854 Y	1.45	0.951
	5	-0.000172 X + 0.13937 Y	1.23	0.953
	6	-0.000256 X + 0.13121 Y	1.27	0.953
	7	-0.000097 X + 0.13377 Y	1.76	0.991
	8	-0.000021 X + 0.13696 Y	0.20	0.962
	9	-0.000041 X + 0.12746 Y	1.41	0.958
Prefecture Equation (Average)			1.27	0.961

4.3. Area Sensitivity Mapping from Optimum Tilt Angle Adjustment of Solar Panels

The closer a place is to the equator, the greater the tendency for the optimum solar panel tilt angle to be 0^o. That is, angle adjustments for solar panel installation become more important as distance from the equator increases. To prove the hypothesis that the charted territory in Indonesia is very sensitive to the optimum angle of solar panels, the optimum angle adjustment for the amount of solar radiation energy (Wh) received was mapped. As shown in Figure 3, the area in Indonesia most sensitive to the solar panel installation at the optimum angle lies at 4^o South latitude.

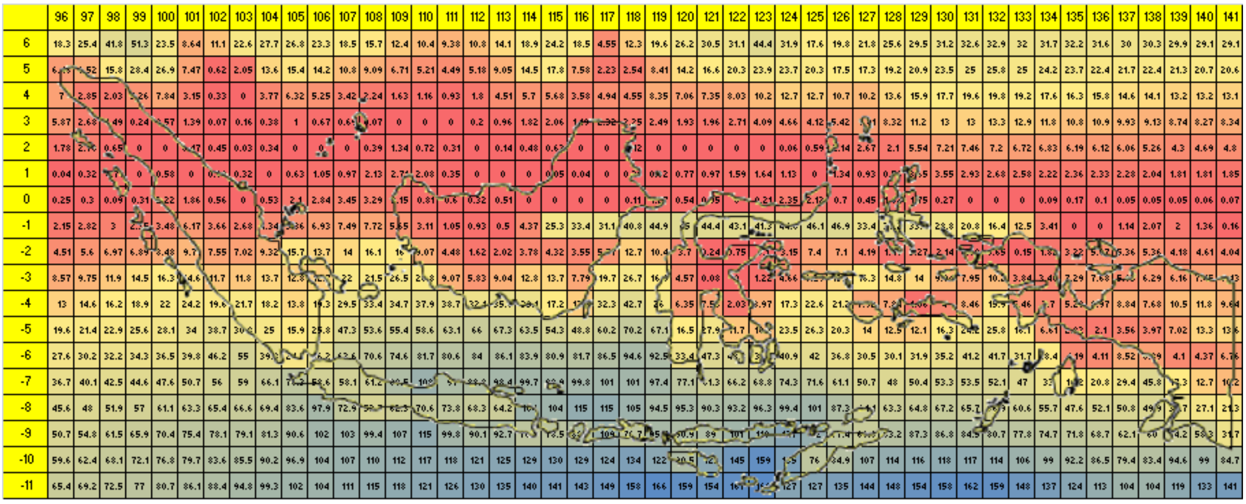

Figure 3 Sensitivity map for tilt angle resulting in optimum radiation on solar panel surface, where X axis is longitude (degrees) and Y axis is latitude (degrees)

4.4. Mapping of the Energy Cost of the Optimum Tilt Angle of Solar Panels in Indonesia

Efforts to determine the optimum tilt angle of solar panels are ultimately meant to improve the economic impact of solar technology. Because solar radiation received by the surface of a solar panel is converted into electricity, investment can be maximized by maximizing the amount of solar radiation received by each panel. In other words, the Cost of Energy (COE) from solar panels can be diminished to improve the economics of solar power plants.

In this study, we mapped the amount of energy produced per year and its related COE. Both variables assume installation at the optimum tilt angle. This study also assumes that polycrystalline solar panels were used. Polycrystalline solar panels are the most widely used around the world, making these reference values more generally applicable. We also assumed a solar capacity of these panels of 1000 kW or 1 MW (U.S. Department of Energy, 2010; www.pvsyst.com).

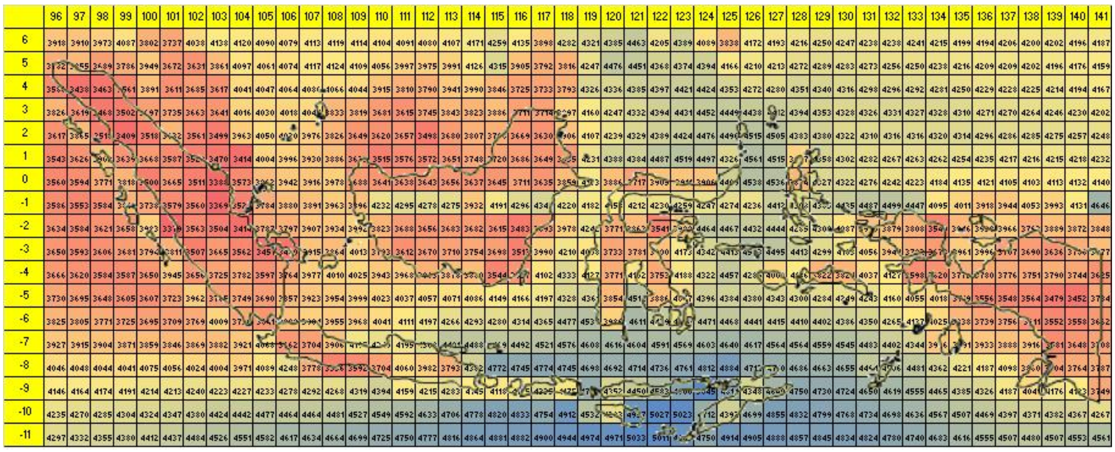

Figure 4 Mapping of electricity production from solar power plants (kWh/day), where X axis is longitude (degrees) and Y axis is latitude (degrees)

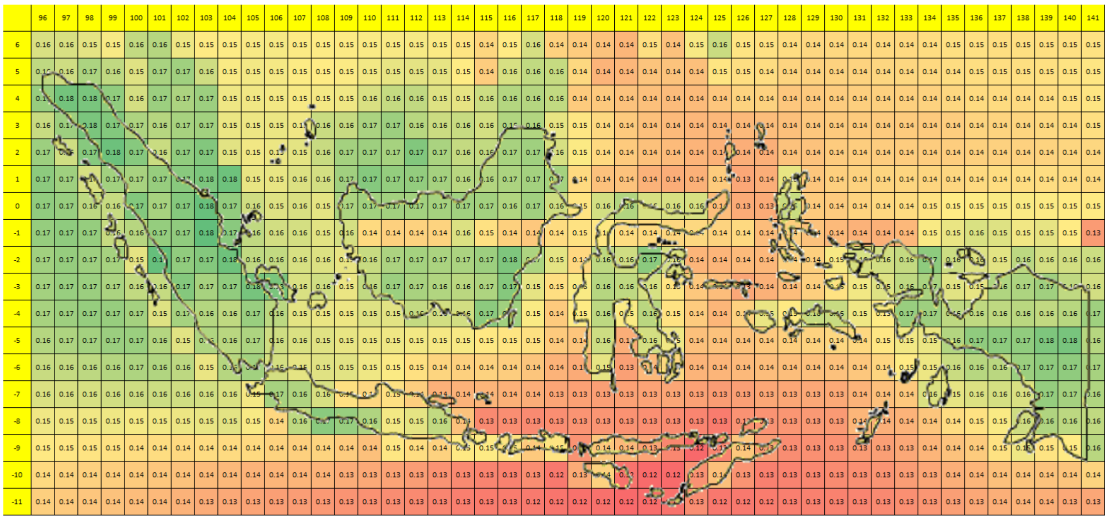

Figure 5 Mapping of COE from solar power plants (USD/kWh), where X axis is longitude (degrees) and Y axis is latitude (degrees)

In Figures 4 and 5, the mapping results show that the lowest COE occurs in locations with the highest solar radiation, when coupled with the adjustment installation of solar panels at the optimum angle. These locations are in the sea, on islands, and in coastal regions. The potential economic benefits to be gained from the installation of solar panels (with a capacity of 1 MW solar and assumed to have a production life of 20 years) at the optimum angle in Indonesia are in the range of US$ 0 to US$ 740,839.66.

4.5. Sensitivity Analysis

The analyses in this study were supplemented by sensitivity analysis. This was done because the assumptions used as variables in calculating the economics of solar projects can shift. For example, there is a trend downward in the price of solar panels, while new technology allows the components of solar panels to have a longer production life, and so forth. To allow for such changes, we conducted sensitivity analysis with the spiderplot method (Sullivan et al., 2006).

We used the spiderplot method because the variables analyzed in this study were numerous, including MARR, useful life, the price of electricity, O&M cost, capacity factor, capacity, and investment cost. If a variable is on the line with the greatest slope, this indicates that when that variable is changed, the overall feasibility of the project will be affected significantly. According to the sensitivity analysis, solar capacity, the price of electricity, the cost of investment, and MARR are the four most sensitive variables. The selling price of electricity (or, in other words, FIT) and external MARR of the project indicate that it can not be pursued without government intervention. Regarding solar capacity, it can be concluded that solar power plants should be built with as large a capacity as possible in order to increase economic feasibility. However, this necessitates greater investment costs at the beginning of a project.

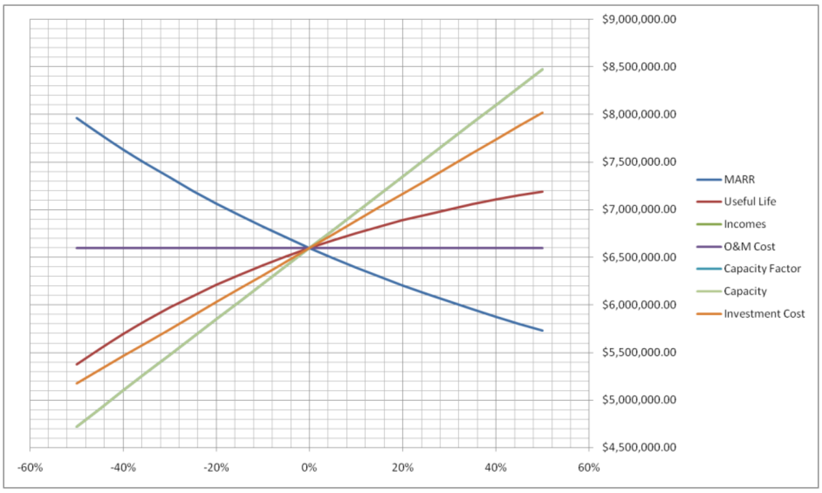

Figure 6 Spiderplot graph for sensitivity analysis, where X axis is percent based on normal value (%) and Y axis is present value (US$)

5. CONCLUSION

The mathematical equations described in this research can be used to determine the optimum tilt angle of solar panels in the Indonesian region. Evidence of correlation and error analysis are summarized. Dividing Indonesia into 9 prefectures resulted in a better error value than using one equation for the entire region. The Indonesian region most sensitive to the installation of solar panels at the optimum angle lies at 4° South latitude. The maximum potential economic benefit that could be gained from the installation of solar panels at the optimum angle in Indonesia is US$ 740,839.66.

6. ACKNOWLEDGEMENT

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