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A COMPREHENSIVE SOLUTION TO AUTOMATED INSPECTION DEVICE SELECTION PROBLEMS USING ELECTRE METHODS

Prasenjit Chatterjee*[1], Suprakash Mondal[2], Shankar Chakraborty[3]

[1]*Department of Mechanical Engineering, MCKV Institute of Engineering, Liluah, Howrah- 711 204, India*
[2]*Department of Production Engineering, Mallabhum Institute of Technology, Bishnupur, Bankura- 722 122, India*
[3]*Department of Production Engineering, Jadavpur University, KOLKATA- 700 032, India*

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ABSTRACT

Selection of an automated inspection device for an explicit industrial application is one of the most challenging problems in the current manufacturing environment. It has become more and more complicated due to increasing complexity, advanced features and facilities that are endlessly being integrated into the devices by different manufacturers. Selection of inspection devices plays a significant role in a manufacturing system for cost effectiveness and improved productivity. This paper focuses on the application of a very popular Multi-Criteria Decision-Making (MCDM) tool, i.e. ELimination and Et Choice Translating REality (ELECTRE) for solving an automated inspection device selection problem in a discrete manufacturing environment. Using a sample case study from the published literature, this paper attempts to show how different variants of the ELECTRE method, namely ELECTRE II, IS, III, IV and TRI can be suitably applied in choosing the most efficient alternative that accounts for both the decision maker's intervention and other technical elements. Using different ELECTRE methods, a list of all the possible choices from the best to the worst suitable devices is obtained while taking into account different selection attributes. The ranking performance of these methods is also compared with that of the past researchers.

Keywords: Automated inspection device selection; Comparative analysis; ELECTRE II, IS, III, IV, TRI; MCDM

1. INTRODUCTION

In the present day's globally competitive manufacturing environment, decision-making for a proper inspection system selection is one of the most challenging tasks for industry. The decision-maker has to select the best alternatives, keeping in mind their relative performance characteristics and also the manufacturing goals and objectives. Increasing demand for a large variety of quality products at lower costs and tough international competition has raised new responsibilities for quality control engineers. Integrated quality control and flexible inspection systems are considered as the most important factor for the integration of quality assurance with automated manufacturing. Golomski (1990) has discussed that an automated inspection system can reduce the indirect cost of inspection, but it increases the depreciation cost and the maintenance cost also. Quality control plays a vital role in the manufacturing industry in order to maintain its competitive edge in the global market.

* Corresponding author's email: prasenjit2007@gmail.com, Tel. +91-33-26549315, Fax. +91-33-26549318
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Quality control is an integral part of modern manufacturing processes, aimed to control the quality of products/parts.

Human inspection is the best possible means for small manufacturing industries. But, compared to large manufacturers, it has deficiencies, such as human fatigue and boredom, inconsistency in the performance and high inspection costs. An alternative to manual inspection is automated inspection. Automated inspection systems can be defined as the automation of one or more of the steps involved in the inspection procedure. An automatic inspection system can increase inspection efficiency and effectiveness, reducing or eliminating human influence. In automated inspection, it is required to automate the different steps involved in the inspection procedure by determining the measuring points, selecting the inspection devices and sensors, planning for a detailed inspection sequence etc. Economic justification of an automated inspection system depends on whether a saving in labour costs and improvement in accuracy will be more than the investment and development cost of the system. Thus, one way to reduce the total manufacturing cost and to provide a more reliable, objective and consistent quality control process is to use an automated inspection system to detect possible defects. The benefits anticipated to be derived from an automated inspection system are improved detection, improved repeatability, a greater number of inspections per shift, electronically retrievable inspection data and improved safety for inspectors in hazardous areas. Choosing an appropriate automated inspection device from available alternatives depends upon the characteristics of the manufacturing system as well as the quality control functions to be integrated. Selection of an appropriate inspection device mainly depends on the characteristics of the manufacturing system and the quality control functions. This requires detailed study and evaluation of so many alternatives in relation to the respective selection criteria. Decision analysis has been recognized as an important tool for the evaluation of major decisions among the scientific community and also in the public sector. Decision-making is the process of sufficiently reducing (rather than eliminating) uncertainty and doubt about the alternatives to allow a reasonable choice to be made. Selection of an automated inspection system involves consideration of multiple feasible alternatives. It is often observed that the selection procedure involves several objectives and it is often necessary to make a compromise among the possible conflicting criteria. For these reasons, MCDM looks at the paradigm in which an individual decision-maker or a group of experts contemplate a choice of action in an uncertain environment. MCDM methods are found to be quite effective to solve such selection problems.

This paper presents a comparative study between the variants of the ELECTRE method, namely, ELECTRE II, IS, III, IV and TRI, while selecting the most appropriate automated inspection system (Pandey & Kengpol, 1995) in a discrete manufacturing environment. A complete ranking preorder derived from all the considered alternative inspection devices, is obtained using the variants of the ELECTRE method and it is observed that the ranking obtained using the ELECTRE-based methods corroborates quite well with the ranking obtained by past researchers, which proves the applicability of the proposed method to solve such types of complex industrial and manufacturing decision-making problems. Two statistical measures are also adopted to compare the relative ranking performances of these methods.

Although, diverse examples from real time manufacturing situations are studied by the past researchers and a number of attempts are made for evaluation, selection and justification of manufacturing strategies, only a few attempts have yet been made to apply any MCDM tool for selecting inspection devices. Even, until this date, considerably less effort has been devoted to studying the relative performance of several MCDM methods employed in a discrete manufacturing environment. This paper takes this opportunity to explore the application feasibility and potentiality of the above-mentioned ELECTRE-based methods to provide more precise and accurate rankings of the alternative inspection devices. In the next section, a detail description of the mathematical formulations of the different ELECTRE methods are presented and discussed.

2. ELIMINATION AND ET CHOICE TRANSLATING REALITY (ELECTRE) METHODS

ELECTRE was envisaged by Bernard Roy (1991) to overcome some deficiencies of popularly used MCDM tools to deal with ordinal attributes without the need for transforming them into cardinal values. ELECTRE is a well known MCDM method that has a history of successful real world applications for its robust ranking technique. It has been applied in various types of decision-making situations. ELECTRE requires an input of criteria evaluations for the alternatives, called the decision matrix which includes preference information expressed as weights, thresholds, and other parameters. All the ELECTRE-type methods involve two major procedures, the modeling of preferences with outranking relations, followed by an exploitation procedure. The detailed procedures of ELECTRE methods are described in the succeeding sections. ELECTRE methods can operate with one or several crispy or fuzzy outranking relations. In fuzzy ELECTRE applications, linguistic preferences can easily be converted to fuzzy numbers. The ELECTRE method is fundamentally based on the multi-attribute utility theory (MAUT) with the intention to improve efficiency without affecting the outcome while considering less information (Cho, 2003). The basic ELECTRE method is a procedure that sequentially reduces the number of alternatives the decision-maker is faced with in a set of non-dominated alternatives. The concept of an outranking relation S is introduced as a binary relation defined on the set of alternatives A. Given the alternatives A_j and A_k, A_j outranks A_k or A_jSA_k, if given all that is known about the two alternatives, there are enough arguments to decide that A_j is at least as good as A_k. The goal of this outranking method is to find all the alternatives that dominate other alternatives while they cannot be dominated by any other alternative. To find the best alternative, the ELECTRE method also requires the knowledge of the weights of all the criteria. Each criterion $C_i \in C$ is assigned a subjective weight w_i (the sum of the weights of all the criteria equals to 1). The steps as involved in different ELECTRE methods are discussed below.

2.1. ELECTRE II

The ELECTRE II method is basically devoted to the ranking problems and the obtained results are in the form of a total ranking preorder among the non-dominated alternatives. The procedural steps as involved in ELECTRE II method are presented below (Hunjak 1997; Milani et al., 2006; Chatterjee et al., 2009; Chatterjee et al., 2010a; Chatterjee et al., 2010b; Chatterjee et al., 2011).

Step 1: Develop the initial decision matrix, X:

$$X = [x_{ij}]_{mxn} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

where x_{ij} is the performance value of i^{th} alternative on j^{th} criterion, *m* is the number of alternatives compared and *n* is the number of criteria.

Step 2: Normalize the original decision matrix X using Equation. (2) to obtain the normalized matrix G. The purpose of normalization is to obtain dimensionless values of different criteria to make them comparable with each other. Several normalization techniques have been proposed by the past researchers to transform the different units into dimensionless values. In ELECTRE-based methods, vector normalization is generally adopted in which each element of the quantified decision-making matrix is divided by its own Euclidian norm. The norm represents the square root of the addition of element value squares, according to each criterion.

$$G = [g_{ij}]_{mxn} = x_{ij} \Big/ \left[\sum_{i=1}^{m} x_{ij}^2 \right]^{1/2} \quad 1 \le i \le m, \; 1 \le j \le n \tag{2}$$

$$\text{where, } x_{ij} = \begin{cases} x_{ij} & \text{for beneficial attribute} \\ 1/x_{ij} & \text{for non-beneficial attribute} \end{cases}$$

Step 3: Determine the weighted normalized decision matrix, D:

$$D = [y_{ij}]_{mxn} = g_{ij} \times w_j \quad (i = 1,2,\ldots,m;\; j = 1,2,\ldots,n) \tag{3}$$

where g_{ij} is the normalized performance value of i^{th} alternative on j^{th} criterion and w_j is the weight of j^{th} criterion.

Step 4: Determine the concordance index c(i,j) for every pair of the alternatives A_i and A_j:

$$c(i,j) = \sum_{g_k(i) \ge g_k(j)} w_k, \quad i, j = 1,2,...,m, i \ne j \tag{4}$$

where $g_k(i)$ and $g_k(j)$ are the normalized measures of performance of the i^{th} and j^{th} alternative respectively with respect to the j^{th} criterion in the decision matrix. Thus, for an ordered pair of alternatives (A_i,A_j), the concordance index c(i,j) is the sum of all the weights for those criteria where the performance score of A_i is at least as that of A_j. Clearly, the concordance index lies between 0 and 1.

Step 5: Compute discordance index d(i,j) as given below:

$$d(i,j) = \begin{cases} 0 \quad \text{if} \quad g_k(A_i) \ge g_k(A_j), & \text{for all k} \\ \frac{1}{\delta} \max\{g_k(A_i) - g_k(A_j)\}, & \text{otherwise} \end{cases} \tag{5}$$

$$\text{where, } \delta = \max\{g_k(A_i) - g_k(A_j)\}$$

Step 6: Once these two indices are estimated, an outranking relation S can be defined as:

$$A_j S A_k \; \textit{if and only if} \; c(j,k) \ge \hat{c} \tag{6}$$

$$\textit{and} \; d(j,k) \le \hat{d} \tag{7}$$

where $\hat{c}$ and $\hat{d}$ are the threshold values as set by the decision-maker. If the threshold values are high, it will be more difficult to pass the tests (normally, $\hat{c} = 0.7$ and $\hat{d} = 0.3$ (Milani et al., 2006). The outranking relation determines the set of non-dominated alternatives. Thus, if alternative A_i outranks alternative A_j, then a directed arc exists from A_i to A_j : $A_i \rightarrow A_j$. For an outranking relation to be judged as true, both the concordance and discordance indices should not violate their corresponding threshold values. When these two tests are performed for all the pairs of alternatives, the preferred alternatives are those which outrank more than being outranked (Figueira et al., 2013).

Step 7: Compute Pure concordance and pure discordance indices as follows:

Pure concordance index $(C_j) = \sum_{k=1}^{n} c(j,k) - \sum_{j=1}^{n} c(k,j) \quad (j \neq k)$ (8)

Pure discordance index $(D_j) = \sum_{k=1}^{n} d(j,k) - \sum_{j=1}^{n} d(k,j) \quad (j \neq k)$ (9)

Once these two indices are estimated, two rankings are obtained on the basis of these two indices and an average ranking is determined from these two rankings. Based on the average ranking, then that alternative is selected which has the best average rank.

2.2. ELECTRE IS

This method is quite similar to ELECTRE II except for the use of fuzzy presentations in criteria values. More precisely, in comparing any two alternatives M_i and M_k with respect to criterion g_j, depending on the difference between the two alternative performances (i.e., $g_j(M_k)$ - $g_j(M_i)$), the concordance index can take a value between 0 and 1. Assuming that g_j is to be maximized, the concordance index can be given by:

$$C_j(M_i, M_k) = \begin{cases} 0, & p_j < g_j(M_k) - g_j(M_i) \\ \dfrac{g(M_i) + p_j - g(M_k)}{p_j - q_j}; & q_j < g_j(M_k - g_j(M_i) \leq p_j \\ 1; & g_j(M_k) - g_j(M_i) \leq q_j \end{cases} \quad (10)$$

Then, similar to ELECTRE I, all the above indices are aggregated in a global concordance index using criteria weights. The discordance indices remain binary (0 or 1). Mi outranks M_k if $C_{ik} \geq$ c and $D_{ik} = 0$ (Shanian & Savadogo, 2009).

2.3. ELECTRE-III

While this method uses the same principles of ELECTRE II, it is similar to ELECTRE IS in the sense that it uses pseudo-criteria instead of classical true-criteria. In order to construct a fuzzy outranking relation in the ELECTRE-III method, three different threshold values, namely, undifferentiated threshold (q_j), strict superior threshold (p_j) and rejection threshold (v_j) are first introduced. For a given random criterion c_j, $0 < q_j < p_j < v_j$. The values of the three thresholds need to be determined according to the practice of concrete problems and the risk attitude of the decision-maker. In this paper, the different threshold values are determined as follows (Liu & Zhang, 2010; Clivillé et al., 2006):

a) **Undifferentiated threshold** (q_j): when the difference between the values of alternative a_i and alternative a_k with respect to criterion c_j is not more than q_j, i.e. when $r_{ij} + q_j \geq r_{kj}$ and $r_{kj} + q_j \geq r_{ij}$, alternative a_i and alternative a_k are considered to be indifferent with respect to criterion c_j.

$$q_j = (\text{maximum attribute value - minimum attribute value})*\text{certain percent} \quad (11)$$

The range of this percentage is usually from 5 to 10, however, it can be regulated appropriately according to the risk attitude of the decision-maker. Here the percent is taken as 10%.

b) **Strict superior threshold** (p_j): when the difference is more than p_j, i.e.when $r_{ij} \geq r_{kj} + p_j$, alternative a_i is considered to be a strict superior to alternative a_k. If $r_{kj} + q_j < r_{ij} \leq r_{kj} + p_j$, then alternative a_i is considered to be a weak superior to alternative a_k.

$$p_j = (\text{undifferentiated threshold } q_j * \text{certain multiple}) \quad (12)$$

The multiple usually ranges from 3 to 10. In this paper, the multiple is taken as 3.

c) **Rejection threshold** (v_j): when the difference between alternative a_k and alternative a_i with respect to criterion c_j is not less than v_j, i.e. when $r_{kj} \geq r_{ij} + v_j$, alternative a_i is not considered to be superior to alternative a_k on the whole.

$$v_j = (\text{maximum attribute value - minimum attribute criterion value})*\text{certain multiple} \quad (13)$$

The multiple ranges from 3 to 5. In this paper, the multiple is taken as 3.

Now, after computing these three threshold values, a single harmoniousness index and a single un-harmoniousness index are computed employing Equations (14) and (15) for each of the alternative pairs.

Single harmoniousness index:

$$SD_j(i,k) = \begin{cases} 1, & \text{if } r_{ij} + q_j \geq r_{kj} \\ 0, & \text{if } r_{ij} + p_j \leq r_{kj}, \\ (r_{kj} - (r_{ij} + p_j))/(q_j - p_j), & \text{otherwise} \end{cases} \quad (14)$$

where SD_j (i,k) represents the degree of supporting the judgment that alternative a_i is superior to alternative a_k in index c_j.

Single un-harmoniousness index:

$$D_j(i,k) = \begin{cases} 0, & \text{if } r_{ij} + p_j \geq r_{kj} \\ 1, & \text{if } r_{ij} + v_j \leq r_{kj}, \\ (r_{kj} - (r_{ij} + p_j))/(v_j - p_j), & \text{otherwise} \end{cases} \quad (15)$$

where $D_j(i,k)$ represents the measure which is rejecting the judgment that alternative a_i is superior to alternative a_k in index c_j.

Once these two indices are estimated, the overall harmoniousness relation and the credit index are determined for all the alternative pairs as follows:

Overall harmoniousness relation:

$$C(i,k)=\frac{\sum_{j=1}^{n} w_j SD_j(i,k)}{\sum_{j=1}^{n} w_j}=\sum_{j=i}^{n} w_j SD_j(i,k), \quad i,k=1,2,...,m. \tag{16}$$

Credit degree index:

$$S(i,k)=\begin{cases} C(i,k), & \text{if } D_j(i,k)\le C(i,k), \forall j \\ C(i,k)\prod_{\{j\in J: D_j(i,k)>C(i,k)\}} \frac{1-D_j(i,k)}{1-C(i,k)}, & \text{otherwise} \end{cases} \tag{17}$$

where S(i,k) represents the measure which supports the judgment that alternative a_i is superior to alternative a_k.

The last step is to calculate a total score and ranking preorder of the alternatives by computing the net advantage values as follows:

$$\delta_k = \sum_{\substack{i=1 \\ i\ne k}}^{m} S(i,k) - \sum_{\substack{i=1 \\ i\ne k}}^{m} S(k,i), k=1,2,...,m. \tag{18}$$

where δ_k represents the net advantage values of the alternatives. The higher the δ_k value, the better is the position of the alternative in the ranking preorder (Papadopoulos & Karagiannidis, 2008).

2.4. ELECTRE IV

In all the above mentioned ELECTRE methods, criteria weights are directly used in the computation of global concordance indices. However, ELECTRE IV is the only version of the ELECTRE methods which does not require the value of criteria weights. Similar to ELECTRE III, this approach also uses a linear fuzzy representation of the criteria for both concordances and discordances. Depending on the magnitude of r_{kj}-r_{ij} , as compared to a set of pre-defined threshold values, the alternative a_i can be strictly, weakly, and hardly preferred over the alternative a_k, or vice versa. Instead of using the value of membership function, the number of criteria falling in each of the above outranking categories is used. Next, a set of credibility degrees similar to ELECTRE III is used to classify the alternatives based on the ascending and descending distillations (Chen, 1997).

2.5. ELECTRE TRI

ELECTRE TRI is a multi-criteria sorting method which assigns alternatives to some predefined categories (Mousseau et al., 2000). In ELECTRE TRI, the preference model is an outranking relation and parameters involved are criteria weights and various thresholds on each of the criteria. The assignment of any alternative results from the comparison with the profiles defining the limits of the categories. Let F denote the set of the indices of the criteria $g_1, g_2 ..., g_m (F = \{1,2,...m\})$ and B the set of indices of the profiles defining $p+1$ categories $(B = \{1,2,..., p\}), b_h$ being the upper limit of category C_h and the lower limit of category is $C_{h+1}, h = 1,2..., p$. ELECTRE TRI builds an outranking relation S which validates or invalidates the assertion aSb_h (and b_hSa), signifying alternative a is at least as good as b_h. Preferences restricted to the significance axis of each criterion are defined through pseudo-criteria. The indifference and preference thresholds $(q_j(b_h))$ and $p_j(b_h)$ constitute the intra-criterion preferential information. This method builds a credibility index the same as ELECTRE III by finding the partial concordance index followed by determining the discordance index followed by the computation of overall harmoniousness relation and the credit index using the equations as used in ELECTRE III method. The cutting level λ is taken as a value in between 0.5000 to 1.0000 for ELECTRE TRI method-based analysis (Lourenco & Costa, 2004). The following two assignment procedures are then followed (Mousseau & Slowinski, 1998).

Conjunctive assignment procedure:

An alternative a will be assigned to the highest category C_h such that aSb_{h-1}

a) Compare alternative a successively to b_i for i = p,p-1,p-2,…..0
b) The limit b_h is the first profile encountered such that aSb_h, assign alternative a to category $C_{h+1} (a \rightarrow C_{h+1})$.

Disjunctive assignment procedure:

An alternative a will be assigned to the lowest category C_h such that $b_h > a$

a) compare alternative a successively to b_i, i = 1,2,…..,p-1
b) b_h being the first profile such that $b_h > a$ assign alternative a to category $C_h (a \rightarrow C_h)$:

3. ILLUSTRATIVE EXAMPLE

In order to demonstrate the aptness of the ELECTRE methods for solving the automated inspection device selection problem, a real time example from Pandey and Kengpol (1995) is considered here. This example deals with the selection of the most appropriate automated inspection device for using in flexible manufacturing systems. Here, eleven criteria and four alternative automated inspection devices are considered. The considered criteria are accuracy (A), volumetric performance (V), repeatability (R), resolution (S), maintainability (M), reliability (L), initial cost (I), operation cost (O), throughput rate (T), environmental factor requirement (E) and flexibility in software interface (F). The four alternative automated inspection devices are CMM (USA), CMM (Japan), AVI (USA) and LASER SCAN (Japan). The quantitative values of the automated inspection device selection criteria are given in Table 1. Among these eleven criteria, A, V, R, S, M, L, T and F are the beneficial attributes where higher values are desirable, and the remaining are the non-beneficial attributes requiring smaller values.

Table 1 Quantitative data for automated inspection system selection problem (Pandey & Kengpol, 1995)

Sl. No.	Inspection device	A	V	R	S	M	L	I	O	T	E	F
1.	CMM (USA)	90	80	80	70	60	85	40	2	70	80	80
2.	CMM (Japan)	80	70	80	70	60	80	30	7	70	80	60
3.	AVI (USA)	60	50	50	80	80	70	20	1	80	60	60
4.	LASER SCAN (Japan)	75	70	70	60	70	70	25	4	80	70	70

Rao (2007) employed the analytic hierarchy process (AHP) to determine the weights of the considered criteria. These same criteria weights, $w_A = 0.2071$, $w_V = 0.0858$, $w_R = 0.2071$, $w_S = 0.0518$, $w_M = 0.0325$, $w_L = 0.0518$, $w_I = 0.0858$, $w_O = 0.0325$, $w_T = 0.1376$, $w_E = 0.0219$ and $w_F = 0.0858$ are used here for the ELECTRE method-based analysis. The best choice of the automated inspection device for the given application is CMM (USA). Pandey & Kengpol (1995) obtained a ranking of the alternative automated inspection devices as CMM (USA) > CMM (Japan) > LASER SCAN (Japan) > AVI (USA) using the preference ranking organization method for enrichment evaluation (PROMETHEE) method. Rao (2007) also obtained the same ranking of the automated inspection devices while solving the problem applying technique for order preference by similarity to the ideal solution (TOPSIS) method.

3.1. ELECTRE II method

At first, this automated inspection device selection problem is solved using ELECTRE II method. For this, the decision matrix, as given in Table 1, is normalized using Equation (2) depending on the nature of the considered attributes. The normalized decision matrix is shown in Table 2.

Table 2 Normalized decision matrix for automated inspection device selection problem

Device	A	V	R	S	M	L	I	O	T	E	F
1	0.5843	0.5850	0.5629	0.4975	0.4411	0.5554	0.0006	0.2500	0.4656	0.0002	0.5882
2	0.5194	0.5119	0.5629	0.4975	0.4411	0.5227	0.0011	0.0204	0.4656	0.0002	0.4411
3	0.3895	0.3656	0.3518	0.5685	0.5882	0.4574	0.0025	1.0000	0.5322	0.0003	0.4411
4	0.4869	0.5119	0.4925	0.4264	0.5147	0.4574	0.0016	0.0625	0.5322	0.0002	0.5147

Now, using Equation (3) the weighted normalized matrix, as given in Table 3, is calculated by multiplying the weights of the respective criteria. The concordance matrix for each pair of inspection devices is then estimated using Equation (4) as given in Table 4. While calculating these concordance indices, if there are ties between the alternatives, they would receive one-half of the criteria weights (Chatterjee et al., 2011). The discordance matrix is computed using Equation (5), as exhibited in Table 5.

Table 3 Weighted normalized matrix

Device	A	V	R	S	M	L	I	O	T	E	F
1	0.121	0.050	0.116	0.025	0.014	0.028	0.028	0.014	0.064	0.009	0.050
2	0.107	0.043	0.116	0.025	0.014	0.027	0.037	0.004	0.064	0.009	0.037
3	0.080	0.031	0.072	0.029	0.019	0.023	0.056	0.028	0.073	0.013	0.037
4	0.100	0.043	0.102	0.022	0.016	0.023	0.044	0.007	0.073	0.011	0.044

Table 4 Concordance matrix

Device	1	2	3	4
1		0.6885	0.6376	0.7544
2	0.3113		0.5947	0.5607
3	0.3621	0.405		0.3192
4	0.2778	0.439	0.439	

Table 5 Discordance matrix

Device	1	2	3	4
1		0.7441	0.6612	1.0000
2	1.0000		0.5332	0.7997
3	1.0000	1.0000		1.0000
4	0.9304	1.0000	0.3999	

Now, the pure concordance and pure discordance indices for the four inspection devices are computed using Equations (8) and (9) respectively, as exhibited in Table 6. From this table, the ranking of the inspection devices is observed as 1 > 2 > 3 > 4. CMM (USA) is the best suited device. CMM (Japan) emerges out as the second best choice and LASER SCAN (Japan) is the worst device.

Table 6 Ranking of alternative devices

Device	Pure concordance index	Initial rank	Pure discordance index	Initial rank	Average rank	Final rank
1	1.1293	1	-0.5252	1	1	1
2	-0.0658	2	-0.4111	2	2	2
3	-0.585	4	1.4058	2	3	3
4	-0.4785	3	-0.4695	1	2	4

3.2. ELECTRE IS method

For application of the ELECTRE IS method for solving this automated inspection device selection problem, the different thresholds values are first estimated using Equations (11), (12) and (13), as shown in Table 7. Concordance and discordance indices for all the alternative pairs are then calculated using Equation (10) and Equation (5) respectively, as given in Tables 8 and 9. Next, the global concordance index, concordance rank and pure discordance indices are computed using Equations (16), (18) and (5) and the corresponding values are given in Table 10. This table also shows the final ranking of the alternative devices. CMM (USA) (Device 1) is the best choice and LASER SCAN (Japan) (Device 4) is the worst choice for an inspection device.

Table 7 Threshold values for inspection device selection attributes

Criteria	A	V	R	S	M	L	I	O	T	E	F
q_j	3	3	3	2	2	1.5	0.0025	0.0857	1	0.0004	2
p_j	9	9	9	6	6	4.5	0.0075	0.2571	3	0.0013	6
v_j	90	90	90	60	60	45	0.075	2.5714	30	0.0125	60

Table 8 Concordance indices for device pairs

Device	1	2	3	4
1		0.9139	0.8602	0.8920
2	0.9672		0.9997	0.8595
3	0.9997	0.9997		0.9997
4	0.9706	0.9997	0.8595	

Table 9 Discordance indices for device pairs

Device	1	2	3	4
1		0.7441	0.6612	1
2	1		0.5332	0.7997
3	1	1		1
4	0.9304	1	0.3999	

Table 10 Global and pure concordance indices of inspection devices

Device	Pure concordance index	Initial rank	Pure discordance index	Initial rank	Average rank	Final rank
1	0.2714	1	-0.5252	1	1	1
2	-0.827	2	-0.4111	2	2	2
3	-1.1399	4	1.4058	2	3	3
4	-0.9706	3	-0.4695	1	2	4

3.3. ELECTRE III method

While applying the ELECTRE III method, the threshold values of Table 7 are used for the calculation of overall harmoniousness index. Values of the overall harmoniousness indices are exactly the same as that of the concordance indices of Table 8, as computed for the ELECTRE IS method. Based on these indices, the Credit degree index for each alternative inspection device pair is estimated using Equation (17) and these values are also exactly same as that of the concordance indices of Table 8. Now, the net advantage values of the alternative devices, as shown in Table 11, are then obtained using Equation (18). After arranging these net advantage values in descending order, the final ranking of the inspection devices is obtained, as exhibited in Table 11. The ranking of the alternative devices is observed as 1 > 2 > 4 > 3 which signifies that Device 1 (CMM (USA)) is the best inspection device, followed by Device 2 (CMM (Japan)), while AVI (USA) is the worst alternative.

Table 11 Net advantage values and final ranks of inspection devices

Device	Net advantage value	Rank
1	0.2714	1
2	-0.827	2
3	-1.1399	4
4	-0.9706	3

3.4. ELECTRE IV method

In the ELECTRE IV method, at first, overall harmoniousness indices are computed using Equation (16) without considering the weights of the attributes. Next, the credit degree index, as shown in Table 12, is calculated for each alternative pair using Equation (17). Table 13 shows the net advantage value and final ranks of the alternative devices obtained using Equation (18). From Table 13, the ranking of the inspection devices is observed as 1 > 4 > 2 > 3. CMM (USA) is the best choice. LASER SCAN (Japan) evolves out as the second best choice and AVI (USA) is the worst choice for a device.

Table 12 Credit degree index

Device	1	2	3	4
1		9.0000	7.0229	8.0000
2	9.0000		10.0000	10.0000
3	10.0000	10.0000		10.0000
4	9.1056	10.0000	7.0000	

Table 13 Net advantage values and final ranks of inspection devices

Device	Net advantage value	Rank
1	4.0826	1
2	-9	3
3	-13	4
4	-6.1056	2

3.5. ELECTRE TRI method

In this method, the credit degree indices are first computed using Equation (16) and these values are exactly same as that of the Concordance indices of Table 8. Now, the conjunctive and disjunctive assignment procedures are followed and the results are presented in Table 14. This table indicates that CMM (USA) and LASER SCAN (Japan) are chosen as the recommended alternatives.

Table 14 Results of the sorting method

Device	Conjunctive category	Disjunctive category
1	1	1
2	3	2
3	3	3
4	1	1

4. COMPARATIVE ANALYSIS

In order to judge the rank conformities among the five variants of the ELECTRE method while solving this automated inspection device selection problem, their ranking performances are now compared using the following two statistical tests (Chatterjee et al., 2014):

(a) Computation of Spearman's rank correlation coefficient (r_s) using Equation (19) is:

$$r_S = 1 - 6\frac{\sum_{i=1}^{m} D_i^2}{m(m^2 - 1)} \tag{19}$$

where, D_i is the difference between ranks R_i and $R^{/}_i$ and *m* is the number of alternatives.

(b) Determination of overall ranking agreement between all the methods using Kendall's coefficient of concordance (z), as shown by Equation (20).

$$z = \frac{\sum_{i=1}^{m}\left(S_i - \frac{\sum_{i=1}^{m} S_i}{m}\right)^2}{\frac{1}{12}k^2\left(m^3 - m\right)} \tag{20}$$

where, *m* denotes number of alternatives and *k* is the number of MCDM methods, and S_i is the sum of ranks assigned to a decision alternative *i* across all *k* MCDM methods. Figure 1 and Table 15 show the comparative ranking preorders of the automated inspection devices as obtained using different MCDM methods.

Table 15 Ranking preorders obtained using different MCDM methods

Device	PROMETHEE (Pandey & Kengpol, 1995)	ELECTRE II	ELECTRE IS	ELECTRE III	ELECTRE IV
1	1	1	1	1	1
2	2	2	2	2	3
3	4	3	3	4	4
4	3	4	4	3	2

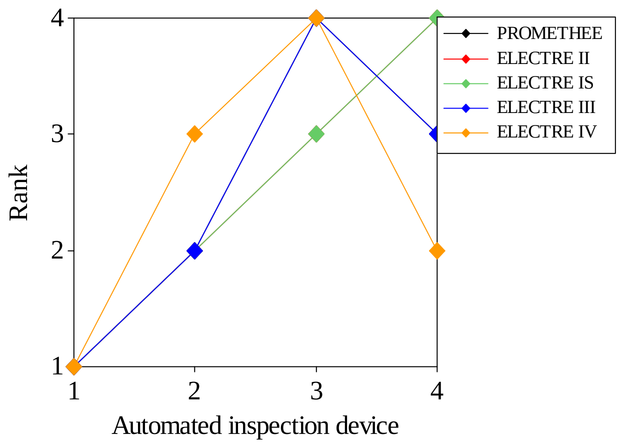

Figure 1 Comparative rankings of automated inspection devices

Using r_s values, the similarity between two sets of rankings can be measured. Usually, r_s value lies between in the range of –1 to +1. The value of +1 denotes a perfect match between two set of rank orderings, whereas, -1 indicates a perfect mismatch between them. Table 16 shows the r_s values when the rankings of the inspection devices as obtained using all the five variants of ELECTRE methods are compared between themselves and also with respect to the rank ordering as derived by Pandey and Kengpol (1995). The r_s value between ELECTRE II and PROMETHEE methods is 0.8000, whereas it is 0.4000 between ELECTRE II and ELECTRE IV methods. Thus, the r_s value ranges between 0.4000 and 1.0000. The rankings obtained using ELECTRE III method exactly match with those as derived by Pandey and Kengpol (1995). Perfect correlation (r_s value of one) exists only between ELECTRE II and ELECTRE IS methods. The performances of other ELECTRE-based methods are also quite satisfactory with respect to r_s value, while the performance of ELECTRE IV method in comparison to other methods is relatively poor which may be due to the fact that ELECTRE IV method does not require the values of criteria weights.

Now, the similarity of rankings obtained by these methods is also measured using *z* value. The *z* value lies between 0 and 1, where a value of 1 is a result of a perfect match (Chatterjee & Chakraborty, 2014). The *z* value for this automated inspection device selection problem is computed as 0.8080, which suggests that there is a very strong agreement between the variants of ELECTRE method. A high *z* value signifies the suitability of these methods to solve the considered automated inspection device selection problem.

Table 16 Spearman‘s rank correlation coefficient values between different methods

Method	ELECTRE II	ELECTRE IS	ELECTRE III	ELECTRE IV
PROMETHEE	0.8000	1.0000	0.8000	0.8000
ELECTRE II		1.0000	0.8000	0.4000
ELECTRE IS			0.8000	0.4000
ELECTRE III				0.8000

5. CONCLUSION

This paper attempts to apply different ELECTRE methods which are comprised of ELECTRE II, III, IV, IS and TRI methods for solving an automated inspection device selection problem. The obtained results show promise for all the applied methods. All the five variants of ELECTRE method are fundamentally based on two distinct phases. In the first phase, outranking relations are developed and in the second phase, these outranking relations are exploited to obtain the ranking preorder of the alternatives. Each ELECTRE method differs from each other mainly in the construction and exploitation procedures. ELECTRE II method is the first of the ELECTRE family which was mainly designed to deal with ranking problems. This method uses a technique based on the construction of embedded outranking relations and is particularly applied when only all the criteria values are expressed in crisp numerical scores. ELECTRE IS uses pseudo-criteria instead of true-criteria values. ELECTRE III was designed to improve ELECTRE II for dealing with impreciseness and uncertainties. In this method the outranking relation can be interpreted as a fuzzy relation. The novelty of this method is the introduction of pseudo-criteria instead of true criteria. The main difference between ELECTRE III and ELECTRE IS methods lies in the fact that both concordance and discordance indices in ELECTRE III are fuzzy, whereas, the same in ELCTRE IS is binary. The ELECTRE IV method is same as ELECTRE III except for the use of relative weights. It is also based on the construction of a set of embedded relations. ELECTRE TRI is designed to categorize a set of alternatives. It is a sorting method where each alternative is considered independently from the others in order to determine the categories to which it seems justified to assign it, by means of comparisons to profiles, norms or references.

Thus, the ELECTRE family deals with the three different types of problems, such as, choice problems (ELECTRE I and ELECTRE IS), ranking problems (ELECTRE II, ELECTRE III and ELECTRE IV) and sorting problems (ELECTRE TRI). Basically there is no generally accepted approach for making a relative comparison among the variants of the ELECTRE method. The determination of which discrete alternative the ELECTRE method is best in itself is an MCDM problem to be solved by the researchers. The comparative analyses results show that dissimilar ranking of the alternatives among ELECTRE II, IS, III, IV, TRI methods may be attributed due to the subjective judgments and the nature of the mathematical modelling as involved with these variants.

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