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INTEGRATING STEEPEST ASCENT FOR THE TAGUCHI EXPERIMENT: A SIMULATION STUDY

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ABSTRACT
Many previous researches conveyed the superiority of Steepest Ascent (SA) method to find the optimal area in Response Surface Methodology (RSM) by shifting the experiment factor level. By using this method, Design of Experiment (DoE) is enabled to shift the factor level gradually in the right track, so that the global optimum can be reached. However, the response variable that is commonly optimized by using RSM cannot fulfill the classical statistics assumption of surface regression model. Taguchi's orthogonal array, as alternative of RSM, gives loose statistics assumptions in performing the analysis. However, Taguchi's orthogonal array has not yet been supported to shift the factor level to an optimum direction. Adopting the procedures of RSM in finding the optimal level combination using SA, integrating SA method in the Taguchi experiment is proposed in this paper. This procedure is applied into a simulated response surface. Then, the performance of this procedure is evaluated based on its direction to reach the optimum solution. The simulation data representing the real case is generated for two factors. Then, the proposed procedure is applied. The result of this simulation study shows that the integrated SA method in the Taguchi experiment successfully found the factor level combination that yields optimum response even though it is not as close as possible as the RSM results.

Keywords: Optimum response; Regression model; Response Surface Methodology; Steepest ascent; Taguchi

1. INTRODUCTION
Taguchi has introduced offline quality control when quality problems can't be solved only by using classical online quality control. Presently, the role of offline quality control has been increasingly important in the quality improvement process. In the quality improvement experiment, the parameter of designing a product (such as identify factors, blocks, levels, DoE, and so on) becomes the initial step causing unconformity of response during the manufacturing process. In this situation, an experimental design method has completed this offline quality control to optimize the industrial tools setting and to obtain the robust parameter design.
Fisher (1890–1962), in Box (1980), Box and Draper (1987), and Stanley (1966), the first originators, introduced the DoE through his book "The Arrangement of Field Experimentress" (1926). They explained that DoE as a method for analyzing experimental result in agricultural field. In these results, the classical DoE such as Completely Randomized Design (CRD), Randomized Block Design (RBD) and also the well-known Factorial Design were mostly applied to help the researcher in investigating single or multifactor effects.

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After a few years, the development of DoE was proposed by Plackett and Burman (1946) and Box and Behnken (1960) who had given multifactor design alternatives with a smaller number of experiment runs.

Furthermore, Box and Wilson (1951) introduced the DoE modification which not only investigated the effect of factors, but also detected the optimal response of multifactor levels by using the Response Surface Methodology (RSM). In the same timeline, RSM dominated the optimization process particularily in industrial machinery-based experiments. In analysing data, both DoE and RSM use mathematical and statistical modeling.

Then, Montgomery (1997), Myers and Montgomery (1995) gave wide explanations about statistical analysing of DoE and RSM for industrial problems. Definitely, it would always deal with classic statistical assumption occurring in mathematical models. However, practioners would find it difficult to use DoE and RSM to solve their problems.

Over time, Taguchi in Belavendram (2001) completed the variation in DoE by proposing a controversial idea. Taguchi, as an engineer, practitioner and statistician, proposed the phenomenal Robust Design concept that did not need to fulfill the classical statistics assumptions. In this concept, Taguchi adopted loss functions for experiment data and used them for the optimization process. Yet, these concepcts, such as The Orthogonal Array design, Signal-to-noise ratio, more simple analysis procedures, and the absence of statistical assumptions, have become the engineer's choice to solve industrial problems.

Currently, the engineers prefer choosing a simple and practical optimization method because of its short optimization time and its reliability in results. However, both the RSM and Taguchi method have the same DoE based, but their applications are complementing or even weakening each other. Both methods should be considered to be integrated, and should be not seen as two competing methods.

2. METHODOLOGY

This paper contains a study of a theoretically non-mathematical experimental design based on the RSM or Taguchi implementation procedures. The discussion is about how to integrate the Taguchi and RSM's steepest ascent procedures will be qualitatively explained, considering both the strengths and weaknesses from each method.

2.1. Response Surface Methodology (RSM)

Mathematically, RSM performs some explanatory variables modelling with one or more response variables. This method uses Fisher's DoE based and the main idea is to determine an optimal point for response variables regarding the factor level setting for the explanatory variables. When RSM is applied in an experiment, then the errors obtained within experiment data cannot be avoided. Therefore, the statistical interpretation is needed. RSM, is only a kind of linear regression that models the relationship between the explanatory and response variables. Implementing the RSM, four steps should be conducted as follows:

a. First step: designing and conducting first order experiment, then modeling it with linear first order regression. First order experiment design consists of simple and small number of experiments, as in Montgomery (1997). The factor levels used for this experiment refer to the current machine operating condition. Usually, two levels are used such as "high" and "low" values.

b. Second step: checking the response surface from the first order design. If there is any lack-of-fit for the regression model, then maybe the optimal solution has been found. Otherwise, if there is no lack-of-fit meaning that the experiment should be continued to search new factor level that can optimize the response

c. Third step: Conducting the steepest ascent (or descent) experiment. The factor levels should be shifted onto the various settings along operational machine condition that refers to the path of steepest ascent. This shifting could be stopped while the optimal response indication has been found.

d. Fourth step: When the indication of optimal response is found at once, the second order design experiment would be conducted with new factor levels that have shifted from the first order design. Then, it models the second order regression. If there is no lack-of-fit in this second order, it means that the optimal response has been found. Then, the optimal factor levels combination could be found by using a simple mathematical derivation of its regression model. Commonly, the optimal factor levels combination comprises unique levels in the middle of the previous "high" and "low" levels.

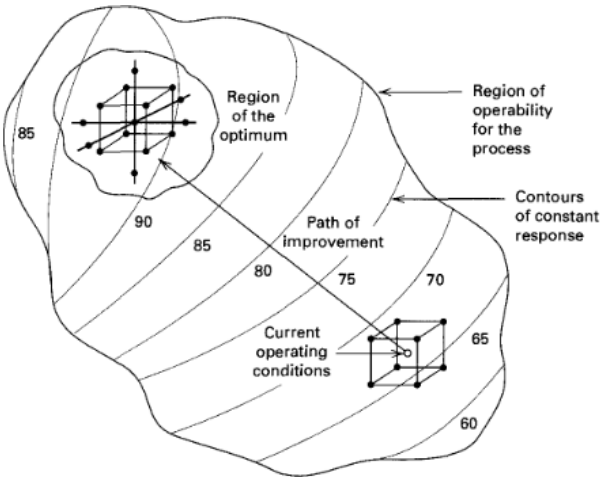

Figure 1 Illustration of steepest ascent path in RSM
(adopted from Montgomery, 1997)

In addition, since the classical statistics assumptions are requirements of the regression models, all of those steps should involve assumption diagnostics for error terms. When the assumptions cannot be complied at once, then the RSM has failed to be interpreted. The ilustration for searching the optimal response is shown in Figure 1.

2.2. The Taguchi Method

Years later, the Taguchi method was presented as an alternative. Adopting the fractional factorial DoE in designing the experiment, Taguchi modifies the design and re-arranges them to other designs called orthogonal arrays. Taguchi promises smaller number of experiments than the DoE model and there is no need to fulfill statistical assumptions, but it still adapts to the optimal response findings. The list of orthogonal arrays can be found in many resources, as in Belavendram (2001) and Roy (1990).

The optimization procedure used in the Taguchi method becomes simpler than the RSM. Taguchi simply uses the response table and graph from the experiment data to be analyzed, and then directly finds the combination of levels that gives the optimal response. Of course, the optimal factor levels are a combination of current levels used in the array's orthogonal experiment. However, the Taguchi method cannot find kinds of the unique levels of the combinations between the "high" and "low" levels as in the RSM, because of the absence of the mathematical derivation in obtaining it. It also only can accommodate not more than two factors interaction.

Another advantage of the Taguchi method is the data transformation into signal-to-noise ratio (SNR) form. Transformed data was claimed to be able to find optimal factor levels combinations that optimized the response, reduced its variance, and event decreased its quality cost all at once (Belavendram, 2001), because it involved a quality loss function suitable for the optimization types, i.e. the nominal, the best, the smaller the better, and the larger the better. This is the reason why the Taguchi method was called a robust design; it is a robust product parameter design that minimizes the variation between products and also is involved in reducing their quality cost, especially in mass production. However, these advantages still bring other weaknesses. The Taguchi method does not accommodate moving the factor levels to the most optimal condition, as the RSM does. It means that the Taguchi method only found the optimal levels combination among the studied factor levels in an orthogonal array, so there was no chance to find kinds of unique value levels. Some practical applications of the Taguchi method in manufacturing field can be found in Rosiawan (2011) and Hadiyat (2011, 2012a, 2012b).

This research proposed to integrate the steepest ascent (or descent) procedure in the Taguchi method. A bivariate normal distribution with some additional error terms was generated to represent the surface form of response. Two factors were involved with certain "high" and "low" level values. Of course, the factors should be in quantitative form, since the steepest ascent path cannot accommodate qualitative ones. The generated response surface data is shown in Figure 2.

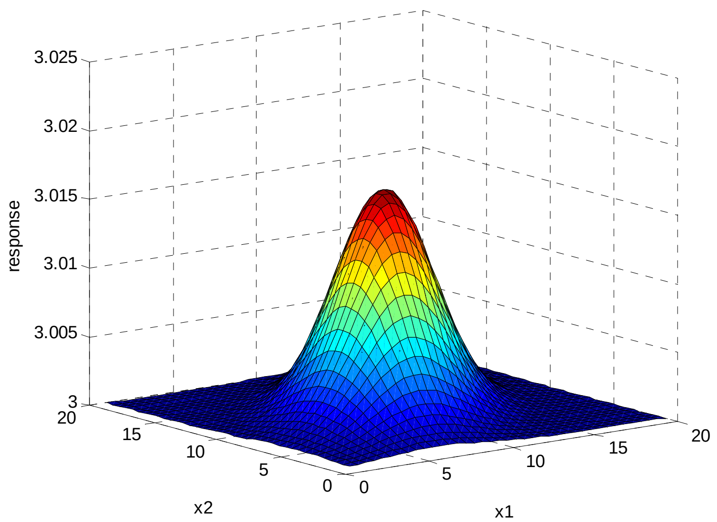

Figure 2 Generated response surface with two factors

The generated response has an optimal value y = 3.0163, with the optimal factor levels combination at x_1 = 10, and x_2 = 10. The starting level for each factor is shown in Table 1, located at the slope. The L_4 orthogonal array was then used to conduct the experiment that involves two factors. The next step is by finding the steepest ascent path that is directed to the top of the response. Final optimization was done by re-designing experiment level around the top response, and then the optimal level combination would be found.

3. RESULTS: INTEGRATED STEEPEST ASCENT IN THE TAGUCHI METHOD

The main reasons for integrating the steepest ascent in the Taguchi experiment are; first of all, the Taguchi method cannot accommodate factor levels moving to obtain the best optimal response. Second, the Taguchi method does not need to involve statistical assumptions in their analysis. Some literature sources performed an analysis of variance for the Taguchi method that obtained data for calculating the contribution ratio of each factor (Belavendram, 2001), and for the mathematically proven significant factors effect (Park, 1996). Integrating the steepest ascent would need to fulfill statistical assumptions only while the regression model was applied to determine the path.

Replacing the first order design in RSM, for the two factors, an L_4 orthogonal array could be chosen. Then, the experiment would be conducted. In this condition, the response of L_4 orthogonal array was obtained that refers to the simulated bivariate normal distribution. The simulated response was shown in Table 1.

Table 1 The Taguchi method first order design (L_4)

Run	X_1	X_2	e	Response	
				Replicate 1	Replicate 2
1	1	1	1	3.00015	3.00014
2	1	2	2	3.00012	3.00015
3	2	1	2	3.00023	3.00022
4	2	2	1	3.00036	3.00037

The values "1" and "2" for X_1 and X_2 represent the "low" and "high" factor levels. The first order regression model for the data in Table 1 was obtained as Equation (1).

$$Y = 2.99987 + 0.00015X_1 + 0.00005X_2 \quad (1)$$

Based on the Equation (1), a contour plot was generated as in Figure 3. Both regression analyses and the contour plot shows that there was no lack-of-fit, in other words, the response was drawn linearly and that no such optimal value occurs between the high and low levels. If the Taguchi method is applied for analyzing the data in Table 1, then the response table could be obtained as in Table 2. The optimal level combination would be level 2 for X_1 and level 2 for X_2.

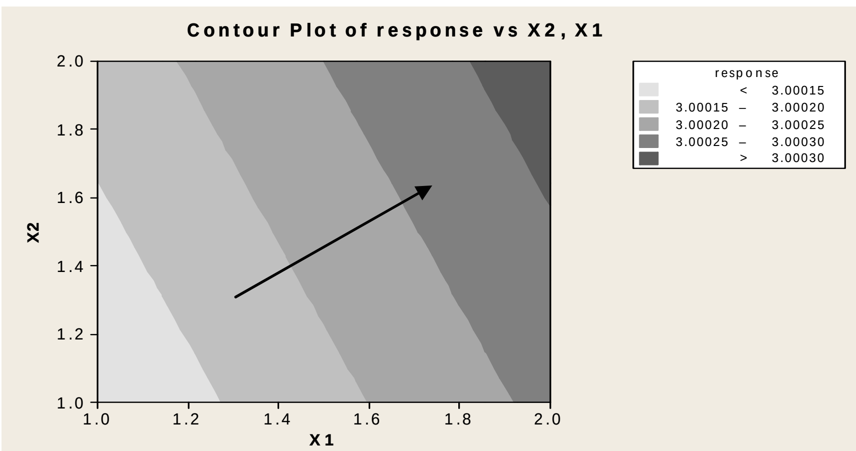

Figure 3 Contour plot for first order design and the path for steepest ascent (represented by arrow)

Table 2 Response table for the Taguchi method analysis

Level	X_1	X_2
1	3.00014	3.00019
2	3.00030	3.00025
delta	0.00016	0.00006

The optimal solution obtained by the Taguchi method did not represent the best optimum response. The reason why the Taguchi method needs to apply factor levels path is for searching through the steepest ascent to find the best solution. As in Montgomery (1997), the path to move the factor level can be calculated as the ratio of both regression coefficients in Equation (1). The coefficients were 0.00015 and 0.00005 for X_1 and X_2 respectively. Then, the path of steepest ascent could be calculated by 0.00005/0.00015 = 0.33, which means that while X_1 level was moved for 1 measurement unit, then X_2 would be moved for 0.33 unit. Table 3 shows the path of steepest ascent and the experiment response regarding the moved factor levels.

Table 3 Path of steepest ascent (moved factor levels)

Steps	X_1	X_2	Experiment Response
Delta (Δ)	1	0.33	3.0005518
Origin	1.5	1.50	3.0001811
origin + Δ	2.5	1.83	3.0002703
origin + 2Δ	3.5	2.16	3.0007254
origin + 3Δ	4.5	2.49	3.0014780
origin + 4Δ	5.5	2.82	3.0021133
origin + 5Δ	6.5	3.15	3.0026532
origin + 6Δ	7.5	3.48	3.0027415
origin + 7Δ	8.5	3.81	3.0031288
origin + 8Δ	9.5	4.14	3.0027265
origin + 9Δ	10.5	4.47	3.0025019
origin + 10Δ	11.5	4.80	3.0018224
origin + 11Δ	12.5	5.13	3.0005518

Once the peak of response have reached as in Figure 4, then the steepest ascent experiment steps could be stopped, and it was indicated that the optimal response was around the peak point. The best optimum response was found around the level value of 8.5 and 3.81 for X_1 and X_2 respectively. It means that that the level of both factors should be moved approaching these values. Then, the previous high and low levels would be replaced with the newly moved level value. Assuming the new level value as shown in Table 4, the final experiment would be conducted regarding the new level value, and the responses were recorded accordingly.

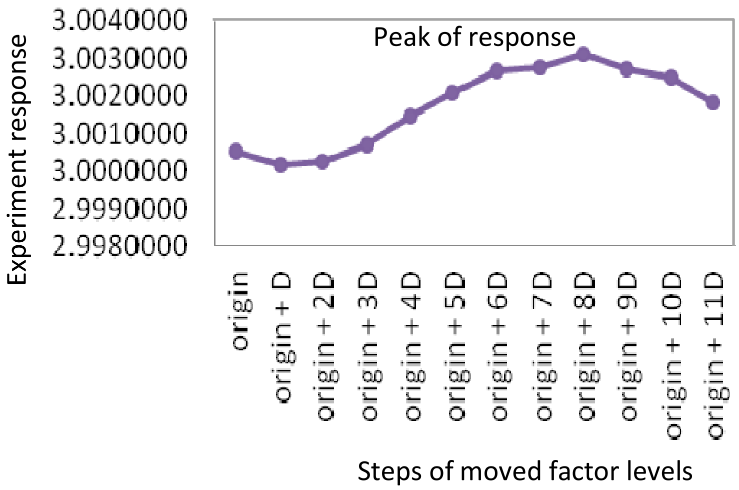

Figure 4 Response variable along path of steepest ascent

Table 4 Response table for final Taguchi experiment analysis

Run	Old X_1	Old X_2	New X_1	New X_2	Response	
					Replicate 1	Replicate 2
1	1	1	8	3.3	3.002436	3.001938
2	1	2	8	4.3	3.003573	3.00341
3	2	1	9	3.3	3.002037	3.00247
4	2	2	9	4.3	3.003813	3.004127

The optimal levels combination would be $X_1 = 9$, and $X_2 = 4.3$. The value of response variable was around 3.0029, far enough from the real optimum response at 3.0163 as mentioned in section 3.

4. DISCUSSION

The proposed steepest ascent is unable to reach the best optimum response, but it still gives a better approach in shifting the factor levels to the closer point for the optimization effort. The main reason is the experimental design RSM uses first order design that has met the statistical requirements, such as the number of experiment runs, and the additional center point at each factor level. Surely, the regression model applied in the first order RSM gives the best result in detecting the path of the steepest ascent. For the final RSM experiment that is known as the Second Order Model, the well-known Central Composite Design must be applied to detect the best optimal response.

Otherwise, the L_4 orthogonal array used by the Taguchi method does not accommodate the center point in the factor levels. However, this design is proposed for finding the best levels combination simply by only calculating the response mean without doing the modelling step at all. Integrating the steepest ascent in the Taguchi experiment, can only be conducted as necessary to shift the initial factor levels onto the optimal factor levels. When a researcher feels satisfied with the Taguchi result in the first order steps, then the steepest ascent would not be needed to be conducted.

5. CONCLUSION

Practically, the Taguchi method gives a better and simpler procedure to find the factor levels combination that optimizes the response. The RSM seems too complicated because of its statistical and mathematical base. Shifting the factor levels to the new value that optimize the response was done by applying the steepest ascent calculation in the Taguchi experiment, even though the result was not as close as the best optimum. However, this research still needs to be developed for the real experiment.

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