

## MODELING OF THE EFFECT OF CORRECTIVE AND PREVENTIVE MAINTENANCE WITH BATHTUB FAILURE INTENSITY

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### ABSTRACT

The aim of this paper is to propose a general model to illustrate the joint effect of corrective and preventive maintenance on repairable systems. The intensity of the failure process without maintenance is characterized in bathtub form. The maintenance effect is expressed by the change induced on the failure intensity before and after maintenance. It takes into account the possibility of dependent maintenance times with different effects. The likelihood functions are derived, so parameter estimations and assessment of the maintenance efficiency are possible. The properties of the parameters estimators have to be theoretically studied. Finally, results are applied to a real maintenance data set.

*Keywords:* Bathtub failure intensity; Estimation; Imperfect maintenance; Reliability; Repairable system

### 1. INTRODUCTION

Many models of the maintenances policies were studied, providing evidence that. Various and varied criteria are used to measure the quality and the efficiency of the maintenance. The majority of the approaches of the maintenance is based on the research of Kwam-Peña (2005) and Peña et al. (2007). A methodology of the maintenance project called maintenance based on reliability was developed firstly by Kijima et al. (1996) and recently by Finkelstein (2008). However, the reliability of the industrial systems depends closely on the efficiency of these maintenance actions. An effective management of the maintenance policy requires a realistic modeling of their effects.

Analytically, the instants of corrective maintenance (CM) are obviously random variables, having occurred on dates which were not envisaged in advance. Because, the dates of the preventive maintenances (PM) are usually fixed before the starting of the system, they are deterministic. Various research tasks relate to the probabilities and the stochastic models of the failure and maintenance process as first studied by Nakagawa (1986), Kijima et al.(1988).

Some authors proposed models which simultaneously account for the two maintenance effects. This type of model was treated with the beginning by Andersen et al. (1993), then by Langseth-Lindqvist (2004) and Doyen-Gaudoin (2004) for the Arithmetic Reduction of Intensity (ARI) and Arithmetic Reduction of Age (ARA) models. These researchers concluded that the PM is deterministic and very few of them proposed models which would make it possible to take into account imperfect maintenances and PM occurrences at the random dates. They also make to integrate the possibility of dependence between the dates of CM and PM.

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We will explain with which limit we can generalize the whole of first models in order to take into account the effect of the two types of maintenance.

We are interested in studying the simultaneous effects of preventive and corrective maintenance, depending on whether the PM programs are selected. We focused our study on PM with planned dates, which are given according to the maintenance dates already observed. Again it can be appropriate for the case of random PM. The basic assumptions are that the PM dates are provided and deterministic, and that they are completely defined in the system startup. The CM-PM process is characterized in this case by the failure intensity and the PM policy.

In this study, we recall various notations:

- $\{Y_k\}_{k \geq 1}$  the waiting duration of the next maintenance when it is preventive
- $\{X_k\}_{k \geq 1}$  the waiting duration of the next maintenance when it is corrective
- $\{U_k\}_{k \geq 1}$  indicators of the maintenance types
- $\{W_k\}_{k \geq 1}$  the durations between maintenances
- $\{D_k\}_{k \geq 1}$  the successive instants of maintenance,  $D_k = \sum_{i=1}^k W_i$
- $\{N_t\}_{t \geq 0}$  the counting process of CM
- $\{T_k\}_{k \geq 1}$  the successive instants of CM,  $T_k = \min\{t \in \mathbb{R}^+ / N_t = k\}$
- $\{M_t\}_{t \geq 0}$  the counting process of PM
- $\{\Gamma_k\}_{k \geq 1}$  the successive instants of PM,  $\Gamma_k = \min\{t \in \mathbb{R}^+ / M_t = k\}$
- $\{\tau_k\}_{k \geq 1}$  the durations between PM,  $\tau_k = \Gamma_k - \Gamma_{k-1}$ ;
- $\{K_t\}_{t \geq 0}$  the counting process of maintenance,  $K_t = \max\{j \in \mathbb{N} / D_k \leq t\}$

The CM and PM processes check respectively that:

$$N_t = \sum_{j=1}^{K_t} U_j \quad \text{and} \quad M_t = \sum_{j=1}^{K_t} (1 - U_j)$$

## 2. CHARACTERISTICS OF THE FAILURES PROCESS

The failure process without maintenance is characterized by a failure intensity defined by the sample principle given in the study of Dijoux (2009) and Krit-Rebai (2011):

$$\lambda(t) = \begin{cases} \frac{1}{\eta_0} + \frac{\beta_1}{\eta_1^{\beta_1}} (t^{\beta_1-1} - \gamma_0^{\beta_1-1}) & \text{si } 0 < t < \gamma_0 \\ \frac{1}{\eta_0} & \text{si } \gamma_0 \leq t \leq \gamma_1 \\ \frac{1}{\eta_0} + \frac{\beta_2}{\eta_2} \left(\frac{t-\gamma_1}{\eta_2}\right)^{\beta_2-1} & \text{si } t > \gamma_1 \end{cases} \quad (1)$$

The survival function is:

$$\mathcal{S}(x, y) = \Pr\{X_1 > x, Y_1 > y\}, \quad \mathcal{S}(t) = \mathcal{S}(t, t),$$

It was shown by Bunea et al. (2002) that whatever the joint law of  $(X, Y)$ , there exists always a couple of independent random variables  $(\tilde{X}, \tilde{Y})$  which get the same law as  $(X, Y)$  for  $(W, U)$ . The information quantity of the data only enables estimation of the sub-survival functions:

$$\forall t \in [\gamma_1, +\infty[, \mathcal{S}_x^*(t) = \Pr\{X > t, X < Y\} = \Pr\{W > t, U = 1\} \quad (2)$$

and

$$\forall t \in [\gamma_1, +\infty[, \mathcal{S}_y^*(t) = \Pr\{Y > t, Y < X\} = \Pr\{W > t, U = 0\} \tag{3}$$

Dorrepaal (1996) proved that  $\mathcal{S}_x^*(t)$  and  $\mathcal{S}_y^*(t)$  form a pair of the sub-survival functions. However, the conditional sub-survival function is one conditioned on the event that the mode of failure is expressed.

The likelihood of data coming from a Poisson process, with general intensity  $\lambda(t)$ , is a function of seven parameters  $\gamma_0, \gamma_1, \eta_0, \eta_1, \eta_2, \beta_1, \beta_2$ , and which is written as follows:

$$\mathcal{L}(\theta; t_1, \dots, t_n) = \left[ \prod_{i=1}^n \lambda_{t_i} \right] \exp \left\{ - \sum_{i=1}^n \int_{t_{i-1}}^{t_i} \lambda_s ds \right\} \tag{4}$$

In this case, intensity has not the same form front  $\gamma_0$ , between  $\gamma_0$  and  $\gamma_1$ , and afterwards  $\gamma_1$ . This direction requires distinguishing the  $\gamma_0$  and  $\gamma_1$  positions, in relation to  $n$  failure dates. With the aim of simplifying calculation,  $\gamma_0$  and  $\gamma_1$  will be often fixed in two failure instants  $t_i$  and  $t_j$  to hold in check  $t_i < t_j$ , particularly in degradation test treatment. For general case, where  $\gamma_0$  and  $\gamma_1$  are unspecified, that gives  $\frac{n(n+1)}{2}$  possible likelihood forms, denoted  $\mathcal{L}_{i,j}$  when one  $i$  observed failures front  $\gamma_0$  and  $j$  failures between  $\gamma_0$  and  $\gamma_1$ . The likelihood function was developed in the same way by Bertholon et al. (2004) and is as follows:

$$\mathcal{L}_{i,j} = \left[ \prod_{k=1}^i \left( \frac{1}{\eta_0} + \frac{\beta_1}{\eta_1^{\beta_1}} \left( t_k^{\beta_1-1} - \gamma_0^{\beta_1-1} \right) \right) \right] \times \left( \frac{1}{\eta_0} \right)^j \times \left[ \prod_{z=j+1}^n \left( \frac{1}{\eta_0} + \frac{\beta_2}{\eta_2} \left( \frac{t_z - \gamma_1}{\eta_2} \right)^{\beta_2-1} \right) \right] \times e^{-\left(\frac{\gamma_0}{\eta_1}\right)^{\beta_1} - \frac{1}{\eta_0} t_n - \left(\frac{t_n - \gamma_1}{\eta_2}\right)^{\beta_2}} \tag{5}$$

with  $1 \leq i \leq n$   
 $i + 1 \leq j \leq n$

### 3. MIXING CORRECTIVE AND PREVENTIVE MAINTENANCE

#### 3.1. The Minimal CM and PM Models

Under the assumption of minimal maintenance, all the effective ages are equal to the date of the last maintenance,  $a_j(\mathcal{K}, \mathcal{U}) = \mathcal{D}_j$ , for all  $j \geq \mathcal{K}_{\gamma_1} + 1$ . This means that the maintenance effects are minimal. following a maintenance action, the system finds its state right before the maintenance action. The intensities of the CM and PM are then checked by:

$$\forall t \geq \gamma_1, \lambda_t^N(t) = \lambda_c(t) \text{ and } \lambda_t^M(t) = \lambda_p(t)$$

In this model the process of CM and PM are two Non-Homogeneous Poisson Processes (NHPP), and their intensities are two random functions. The corresponding likelihood function is expressed while using the Equation (5):

$$\mathcal{L}_{i,z,t}(\theta) = \left[ \prod_{k=1}^i \left( \frac{1}{\eta_0} + \frac{\beta_1}{\eta_1^{\beta_1}} \left( t_k^{\beta_1-1} - \gamma_0^{\beta_1-1} \right) \right) \right] \left( \frac{1}{\eta_0} \right)^z \exp \left\{ - \left( \frac{\gamma_0}{\eta_1} \right)^{\beta_1} - \frac{1}{\eta_0} t_{\mathcal{K}_t} \right\} \left[ \prod_{j=\mathcal{K}_{\gamma_1}}^{\mathcal{K}_t} \frac{[S_x^*(\mathcal{D}_j)]^{1-u_j} [S_y^*(\mathcal{D}_j)]^{u_j}}{S(\mathcal{D}_j)} \right] \mathcal{S}(t) \tag{6}$$

This last function is equivalent to the likelihood function developed in Doyen (2010). This function is not simply because it composed by two unspecified and independent NHPP. In fact, the intensities of these processes can not be it and can have common factors. A type of behavior which does not arise for the perfect CM and PM models.

**3.2. The Perfect CM and PM Models**

In these models if all the effective ages are null,  $a_j(\mathcal{K}, \mathcal{U}) = 0$ , for all  $j \geq \mathcal{K}_{\gamma_1}$ , then, we can obtain the perfect corrective and preventive maintenances. After an action of maintenance, the system is renewed in its new state. This case is answered with the traditional approach of Competing Risk used by Bedford and Mesina (2000), by Cooke and Bedford (2002), and more recently by Doyen and Gaudoin (2006) in which, the intensities of the CM and PM processes are checked by:

$$\forall t \geq \gamma_1, \lambda_t^N(t) = \lambda_c(t - \mathcal{D}_{\mathcal{K}_t}), \lambda_t^M(t) = \lambda_p(t - \mathcal{D}_{\mathcal{K}_t})$$

where  $\lambda_c$  and  $\lambda_p$  indicate respectively the failure and maintenance intensity functions.

And the global maintenance intensity is written:

$$\forall t \geq \gamma_1, \lambda_t^{\mathcal{K}}(\mathcal{K}, \mathcal{U}) = \lambda(t - \mathcal{D}_{\mathcal{K}_t}) \tag{7}$$

Thus, the global maintenance process is a self-excited specific process. More precisely, it is a renewal process. Consequently, the likelihood function is calculated as defined by the Equation (5):

$$\mathcal{L}_{i,z,t}(\theta) = \left[ \prod_{k=1}^i \left( \frac{1}{\eta_0} + \frac{\beta_1}{\eta_1^{\beta_1}} \left( t_k^{\beta_1-1} - \gamma_0^{\beta_1-1} \right) \right) \right] \left( \frac{1}{\eta_0} \right)^z \exp \left\{ - \left( \frac{\gamma_0}{\eta_1} \right)^{\beta_1} - \frac{1}{\eta_0} t_{\mathcal{K}_t} \right\} \left[ \prod_{j=\mathcal{K}_{\gamma_1}}^{\mathcal{K}_t} [\mathcal{S}_x^*(\mathcal{W}_j)]^{1-u_j} [\mathcal{S}_y^*(\mathcal{W}_j)]^{u_j} \right] \mathcal{S}(t - \mathcal{D}_{\mathcal{K}_t}) \tag{8}$$

**3.3. The  $ARA_1$  CM And PM Models**

In the same way using the efficiency models only for the CM, Jack (1998) could define a model generalized by the  $ARA_m$  and  $ARA_\infty$  models. The modeling principle in this case, consists in supposing that the maintenance effect is to reduce proportionally the virtual age to the age supplement accumulated since the last maintenance action. This model corresponds to the MC and MP intensities as follows:

$$\forall t \geq \gamma_1, \lambda_t^N(\mathcal{K}, \mathcal{U}) = \lambda_c \left( t - \sum_{j=\mathcal{K}_{\gamma_1}}^{\mathcal{K}_t} \rho_c^{1-u_{j+1}} \rho_p^{u_{j+1}} \mathcal{W}_{j+1} \right) \tag{9}$$

and

$$\forall t \geq \gamma_1, \lambda_t^M(\mathcal{K}, \mathcal{U}) = \lambda_p \left( t - \sum_{j=\mathcal{K}_{\gamma_1}}^{\mathcal{K}_t} \rho_c^{1-u_{j+1}} \rho_p^{u_{j+1}} \mathcal{W}_{j+1} \right) \tag{10}$$

It is in this context that Langseth-Lindqvist (2004) and Lindqvist (1999) proposed a model which can be regarded as a particular case of the generalized model. The idea of the latter model is equivalent so that each maintenance action has a renewal effect on its own variable of competing risk. Otherwise, each maintenance action has a maximal effect on its own variable of competing risk and a minimal effect on the variable of competing risk corresponding to the other type of maintenance. The MC and MP intensities which are appropriate for this model are expressed by:

$$\forall t, x \geq \gamma_1, \lambda_t^{\mathcal{N}}(\mathcal{K}, \mathcal{U}) = \frac{\left[ \frac{\partial}{\partial x} S(x - \mathcal{J}_{\mathcal{N}_t}, y - \Gamma_{\mathcal{M}_t}) \right]_{(t,t)}}{S(t - \mathcal{J}_{\mathcal{N}_t}, t - \Gamma_{\mathcal{M}_t})} \tag{11}$$

and

$$\forall t, y \geq \gamma_1, \lambda_t^{\mathcal{M}}(\mathcal{K}, \mathcal{U}) = \frac{\left[ \frac{\partial}{\partial y} S(x - \mathcal{J}_{\mathcal{N}_t}, y - \Gamma_{\mathcal{M}_t}) \right]_{(t,t)}}{S(t - \mathcal{J}_{\mathcal{N}_t}, t - \Gamma_{\mathcal{M}_t})} \tag{12}$$

Langseth and Lindqvist (2004) proved in their work that the joint law of the competing risks is identifiable. However, these models are very complex and the risk of assumptions which do not have any practical direction remains a problem.

#### 4. APPLICATION TO REAL DATA

With the aim of testing the validation of the various reformulated models presented in this study, we applied them to real data. The data was gathered from a manufacturing unit of the French Electricity Company, used in a study by Doyen and Gaudoin (2005), and recently by Doyen and Gaudoin (2011). The data is based on 17 units, from both the form of maintenance dates, as well as the type of these maintenances. Our analysis of the data base showed that the service life phase of the system is short. This implies that the intrinsic behavior of the system in absence of maintenance is a superposition of the Weibull type on the improvement and degradation phases.

In the first model we suppose that the CM is minimal and the PM is with  $ARA_1$  type, with a maintenance efficiency factor  $\rho$ . In this case, the likelihood function is written in the same way with the study of Doyen and Gaudoin (2004):

$$\begin{aligned} \mathcal{L}_n(\theta) = & \prod_{k=1}^{\mathcal{K}_\gamma} \left( \frac{\beta_1}{\eta_1^{\beta_1}} (\mathcal{D}_k^{\beta_1-1} - \gamma^{\beta_1-1}) \right) \prod_{z=\mathcal{K}_\gamma+1}^n \left( \frac{\beta_2}{\eta_2} \left( \frac{\mathcal{D}_z - \gamma}{\eta_2} \right)^{\beta_2-1} \right)^{1-u_z} \left( \frac{\beta_2}{\eta_2} \left( \frac{(\mathcal{D}_z - \gamma)(1-\rho)}{\eta_2} \right)^{\beta_2-1} \right)^{u_z} \\ & \exp \left\{ - \sum_{z=1}^{\mathcal{K}_\gamma} \left( \frac{\mathcal{D}_z}{\eta_1} \right)^{\beta_1} - \left( \frac{\gamma}{\eta_1} \right)^{\beta_1-1} \mathcal{D}_z - \sum_{z=\mathcal{K}_\gamma+1}^n \left( \left( \frac{\mathcal{D}_z - \gamma}{\eta_2} \right)^{\beta_2} \right)^{1-u_z} \left( \left( \frac{(\mathcal{D}_z - \gamma)(1-\rho)}{\eta_2} \right)^{\beta_2} \right)^{u_z} \right\} \end{aligned}$$

The logarithm maximization of this function enables calculation to the parameters estimates:

$$\begin{aligned} \hat{\eta}_1 &= 30.312; & \hat{\beta}_1 &= 0.746; & \hat{\eta}_2 &= 29.796 \\ \hat{\beta}_2 &= 2.045; & \hat{\gamma} &= 915.618; & \hat{\rho} &= 0.571 \end{aligned} \tag{13}$$

We noticed that the estimators of the scale parameters  $\eta_1$  and  $\eta_2$  are very close. Moreover, by using the significativity test of difference in averages, we prove that the true values of  $\hat{\eta}_1$  and  $\hat{\eta}_2$  are significantly equal. This means, in practical terms, that the two parts of the curve turn their concavities in the same direction and order. The estimated value of  $\beta_1$  is lower than 1, which means that the first restriction of the failure intensity is decreasing. Therefore, the system tends to improve in the course of time, even in the absence of maintenance. On the other hand, the estimator of  $\beta_2$  is higher than 2, meaning that the second restriction of the failure intensity is increasing. Therefore, the system tends to be degraded in the time course.

An estimated value of the efficiency factor equal to 0.571 implies that the PM reduces the age supplement accumulated since the last action of PM by 57.1%. Figure 1 represents a trajectory of the failure intensity in bathtub form of the  $ARA_1$  reformulated model having parameters like

the values estimated in Equation (13). The PM effect during the degradation phase is as follows:  $0 < \rho < 1$ , which is regarded as imperfect. In this figure the CM and PM dates are represented respectively by stars and circles on the x-axis. In this case, the system is degraded less quickly than right before the maintenance action, and its failure intensity is weaker than right before maintenance.

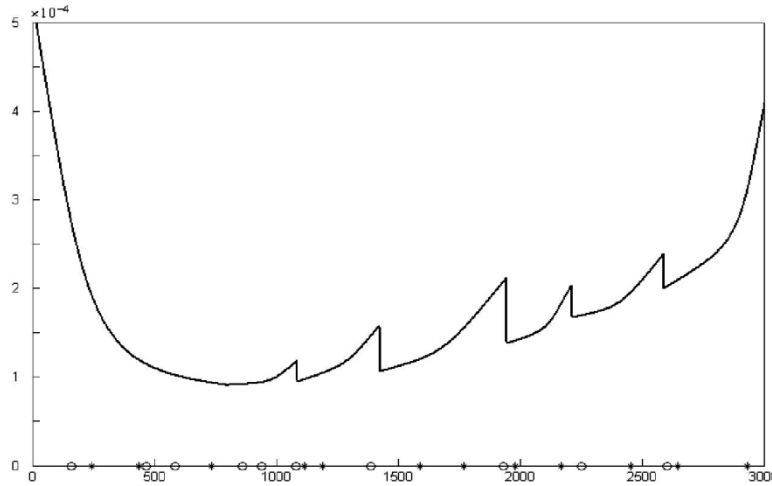


Figure 1 Intensity relating to the estimators in Equation (13)

The second model considers that the CM and PM are both of the  $ARA_m$  type with different efficiency maintenance factors,  $\rho_c$  and  $\rho_p$ . The failure intensity function of this model is complex, as is its likelihood function.

$$\lambda(t) = \begin{cases} \frac{\beta_1}{\eta_1^{\beta_1}} (t^{\beta_1-1} - \gamma^{\beta_1-1}) & \text{if } 0 < t < \gamma \\ \frac{\beta_2}{\eta_2} \left( (t - \gamma - \sum_{i=\mathcal{K}_{\gamma_1}}^{\min\{m-1, \mathcal{K}_t\}} \rho_c^{1-u_i} \rho_p^{u_i} \mathcal{W}_i) / \eta_2 \right)^{\beta_2-1} & \text{if } t \geq \gamma \end{cases} \quad (14)$$

The estimates obtained are given by:

$$\begin{aligned} \hat{\eta}_1 &= 29.145; & \hat{\beta}_1 &= 0.667; & \hat{\eta}_2 &= 48.281 \\ \hat{\beta}_2 &= 1.629; & \hat{\rho}_c &= 0.142; & \hat{\rho}_p &= 0.996 \end{aligned} \quad (15)$$

We notices that the estimate of the scale parameters did not have significant modifications from one model to another. Therefore, this modification can be made by changing the data base, which is more logical and more practical. For the various models, the estimate of the form parameter  $\beta_1$  did not have a significant modification. However, the maintenance actions primarily the PM, decreased the degradation speed of the system. The effect of these actions is to involved the concavity of the failure intensity (since  $\beta_2 < 2$ ). Thus, the system is degraded less and less quickly. According to these last results, the CM seem to have only little effect on the system while intervening to reduce with 14.2 % the total virtual age. At the same time, the PM effect is considered almost perfect. The estimate of the efficiency parameters thus proves coincide with what we observed.

The failure intensities obtained by simulations with the parameters considered, given by the Equation (15), are represented graphically by Figure 2. The axis on the left shows the sudden

system during its degradation period, initially a CM then PM, and the one the right shows the reverse.

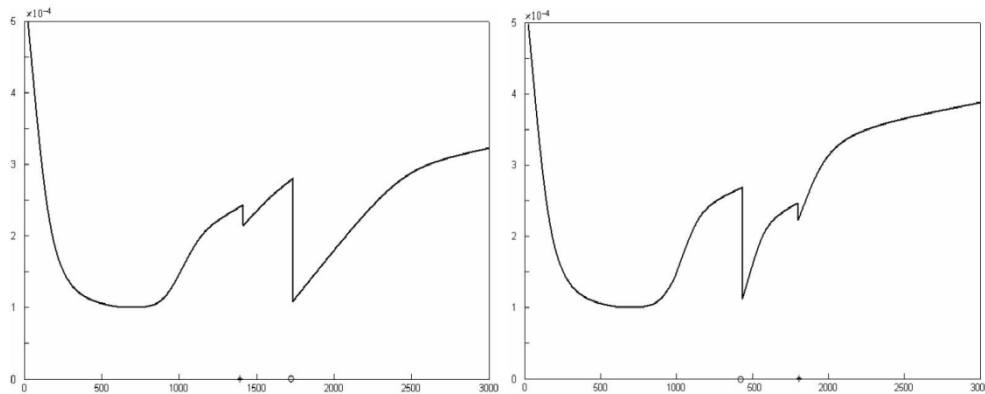


Figure 2 Failure intensity for the model with  $ARA_m$  CM and PM

On these curves, we see that the CM minimally increases the system reliability, whereas the PM strongly increases it. In the same way, we note that the graph beyond the maintenance date is concave. This implies a lower degradation speed than the other models.

It is noted that for all these last models, the estimates of the parameters  $\eta_1, \beta_1, \eta_2$  and  $\beta_2$  are practically of the same order. They do not present statistically significant modification. However, we note that the first restriction of the initial failure intensity is decreasing, considering the estimator of  $\beta_1$  is lower than 1. Thus, this behavior is characteristic of the youth period of the system. In the majority of cases, the estimate of  $\beta_2$  is between 1 and 2. This implies that the second restriction of the curve representative of the initial failure intensity is concave. These two last shapes of curve are distinguished in the Figure 2.

In the first studies of models, we supposed that the CM and the PM effects are identical. The efficiency parameter was estimated by 0.571. This value roughly represents a kind of arithmetic mean between the values considered, 0.142 and 0.996, obtained in the last model.

We used the same data base to test the validity of our reformulation of the Langseth-Lindqvist model which seems to be a conditional PM model. Thus, we took as initial intensities those given by the Equations (16) and (17), with the Weibull type function for the failure rate,  $\lambda_x = \frac{\beta_2}{\eta_2} \left(\frac{t-\gamma}{\eta_2}\right)^{\beta_2-1}$ . Therefore, it is about the bi-varied model with maintenance effects with the  $ARA_1$  type, whose intensities are given by the Equations (9) and (10). Under these conditions, the likelihood function is then given by the following relationship as it was used in Kvam et al. (2002):

$$\mathcal{L}_n(\theta) = \left[ \prod_{k=1}^{\mathcal{K}_{\gamma_1}} \left( \frac{\beta_1}{\eta_1^{\beta_1}} \left( t_k^{\beta_1-1} - \gamma^{\beta_1-1} \right) \right) \right] \exp \left\{ - \left( \frac{\gamma}{\eta_1} \right)^{\beta_1} \right\} \\ \left[ \prod_{z=j+1}^{\mathcal{N}_t} \lambda_{c, \tau_z} (\mathcal{W}_{\mathcal{K}_{\tau_{z-1}}}, \mathcal{U}_{\mathcal{K}_{\tau_{z-1}}}) \right] \exp \left\{ - \sum_{z=j+1}^{\mathcal{K}_t+1} \int_{\mathcal{D}_{z-1}}^{\mathcal{D}_z} \lambda_{c,s} (\mathcal{W}_{z-1}, \mathcal{U}_{z-1}) ds \right\} \\ \left[ \prod_{k=1}^{\square_t} \lambda_{p, \tau_k} (\mathcal{W}_{\mathcal{K}_{\tau_{k-1}}}, \mathcal{U}_{\mathcal{K}_{\tau_{k-1}}}) \right] \exp \left\{ - \sum_{k=\mathcal{K}_{\gamma_1}+1}^{\mathcal{K}_t+1} \int_{\mathcal{D}_{k-1}}^{\mathcal{D}_k} \lambda_{p,s} (\mathcal{W}_{k-1}, \mathcal{U}_{k-1}) ds \right\}$$

$$\text{with, } \lambda_{c,k}(\mathcal{W}_k, \mathcal{U}_k) = \frac{(1-q)\frac{\beta_2}{\eta_2}\left(\frac{T_k-\gamma}{\eta_2}\right)^{\beta_2-1} \exp\left\{-\left(\frac{T_k-\gamma}{\eta_2}\right)^{\beta_2}\right\}}{\exp\left\{-\left(\frac{T_k-\gamma}{\eta_2}\right)^{\beta_2}\right\} - q\left(\frac{T_k-\gamma}{\eta_2}\right)^{\beta_2} I_{\exp}\left(\left(\frac{T_k-\gamma}{\eta_2}\right)^{\beta_2}\right)} \quad (16)$$

and

$$\lambda_{p,z}(\mathcal{W}_z, \mathcal{U}_z) = \frac{q\frac{\beta_2}{\eta_2}\left(\frac{\tau_z-\gamma}{\eta_2}\right)^{\beta_2-1} I_{\exp}\left(\left(\frac{\tau_z-\gamma}{\eta_2}\right)^{\beta_2}\right)}{\exp\left\{-\left(\frac{\tau_z-\gamma}{\eta_2}\right)^{\beta_2}\right\} - q\left(\frac{\tau_z-\gamma}{\eta_2}\right)^{\beta_2} I_{\exp}\left(\left(\frac{\tau_z-\gamma}{\eta_2}\right)^{\beta_2}\right)} \quad (17)$$

The estimates obtained by the maximization of the Log-likelihood function for this model are as follows:

$$\begin{aligned} \hat{\eta}_1 &= 22.698; & \hat{\beta}_1 &= 0.624; & \hat{\eta}_2 &= 37.186 \\ \hat{\beta}_2 &= 1.814; & \hat{\rho}_c &= 0.027; & \hat{\rho}_p &= 0.985; & \hat{q} &= 0.525 \end{aligned} \quad (18)$$

We noted that the estimates of the various parameters (except for  $q$ ), are in the same magnitude of orders as the assumptions previously distinguished. In both cases, the parameters  $\eta_1$ ,  $\beta_1$ ,  $\eta_2$  and  $\beta_2$  have an identical significance. Indeed, they indicate the failure rate parameters of the system, over both the improvement and degradation periods without maintenance. Conversely, the dependent structure between the possible dates of CM and PM is completely different according to the PM type. Obviously, it is astonishing to arrive at comparable estimated values. With regard to the estimated parameter  $\hat{q}$ , represents the probability that a maintenance action is preventive rather than corrective. In the same way, it also represents the approximate proportion of PM compared to the total number of maintenances occurring during the degradation period ( $10/19 = 0.53$ ).

## 5. CONCLUSION

This research deals with stochastic modeling and the parametric estimation of corrective and preventive maintenance efficiencies for industrial systems in degradation. The assessment criterion of the maintenance efficiency is supported with the failure intensity in the bathtub form.

In the first stage, the study is devoted to the maintenance efficiency models already suggested in literature. Thereafter, the study concentrates on models that may make it possible to study the dependence between corrective and preventive maintenances.

The likelihood functions of the models have been derived. Ultimately, a method for assessing the efficiency relating to the total process of corrective and preventive maintenances can be developed and illustrated on a basis of real data issued from industry.

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