Supplementary Material

Mathematical Formulation

Indices:

i, j: vesels' indices, $i, j \in V, i \neq j, V$: set of vessels

b: berth sections index, $b \in B$, B: set of berth sections.

k: quay cranes (QCs) index, $k \in Q$, Q: set of quay cranes

t: time steps index, $t \in T$, which is $T = H + E$

H: the number of equal time steps used to express the planning horizon period.

E: a maximum handling time for all vessels that are served.

s: a discrete scenario index, $s \in S$, S: set of upcoming scenarios.

Model parameters:

L: length of the wharf in meters.

s: the length of a berth section in meters.

l_i: The number of berth sections represents the length of the vessel i, including the safety space between two neighboring mooring vessels, $i \in V$.

b_i^o: the vessel's expected berthing location, $i \in V$.

$[a_i^f, b_i^f]$: a vessel's feasible handling time interval, $i \in V$ and $a_i^f, b_i^f \in T |\ t \leq H$.

$[a_i^e, b_i^e]$: a vessel's expected handling time interval, $i \in V$ and $a_i^e, b_i^e \in T |\ t \leq H$.

m_i: a vessel's total workload in terms of QC–time steps needed for container loading and discharging activities, $i \in V$.

q_i^{min}: The minimum number of QCs should be allocated to a vessel i; $i \in V$.

q_i^{max}: maximum number of QCs that can be assigned to serve a vessel i; $i \in V$.

q_i: the number of QCs that can be assigned to a vessel i, $i \in V$, where $q_i \in \{q_i^{min}, \cdots, q_i^{max}\}$.

α: The QCs' exponential interference coefficient. The allocation of q QCs to a vessel for each time-step results in q^α effective QC–time steps, where $0 < \alpha \leq 1$.

β: The increased coefficient in QC capacity demand due to berthing location discrepancy from the expected berthing location, where $0 \leq \beta \leq 1$

p_s: scenario's probability of being realized, $s \in S$.

m_{is}: capacity demand to service a vessel i in scenario s expressed in QC-time steps, $i \in V, s \in S$.

a_{is}: arrival time of vessel i in scenario s, $i \in V$, $s \in S$.

c_{1i}: penalty cost for waiting of mooring start time of vessel i from its expected start handling time or delay of completion time of vessel i from its designed departure time; $i \in V$.

c_2: QC operational cost rate per time step.

c_3: setup cost of additional QC assigned to the recovery plan.

M: a positive number with a large significant value.

Decision variables:

α_i: the beginning of the vessel's berthing time in the baseline plan, $i \in V$.

β_i: the vessel's end of berthing time in the baseline plan, $i \in V$.

τ_i^{a+}: waiting time before the vessel i begins mooring in the baseline plan, $i \in V$.

τ_i^{a-}: the earliness of mooring start time of vessel i in the baseline plan, $i \in V$.

τ_i^{b+}: tardiness of vessel i 's mooring finish time in the baseline plan, $i \in V$.

τ_i^{b-}: the earliness of mooring finish time of vessel i in the baseline plan, $i \in V$.
b_i: berthing location for a vessel i in the baseline plan, $i \in V$, $b_i \in [1, \dots, L]$.
s_b: berth section b's start time has been occupied; $b \in B$.
e_b: The berth section b finish time has been occupied; $b \in B$.
Δb_i: the discrepancy between the expected berthing location, b_i^o, and berthing location of vessel i at berth section b_i in the baseline plan, $i \in V$.
ω_{ib}: if the center of vessel length i is located at berth section b, set to 1; otherwise, set to 0, $i \in V, b \in B$.
ξ_{ib}: if vessel i is occupying berth section b, set to 1; otherwise, set to 0, $i \in V, b \in B$.
π_{it}: set to 1, if vessel i's berthing time starts at time step $t = \alpha_i$, otherwise set to 0, $i \in V, t \in T |\ t \leq H$.
φ_{it}: set to 1, if vessel i's buffer time finishes at time step $t \geq \beta_i$, otherwise set to 0, $i \in V, t \in T |\ t \leq H$.
θ_{it}: set to 1, if vessel i is mooring at time step t in the baseline plan; $i \in V, t \in T$.
δ_{ij}^x: set to 1 if vessel i's handling time is completed before vessel j's handling time begins, and 0 otherwise.; $i, j \in V, i \neq j$.
δ_{ij}^y: set to 1 if vessel i's berthing location along the quay wharf area is on the left side of vessel j, and 0 otherwise.; $i, j \in V, i \neq j$.
X_{it}^q: set to 1, if vessel i is served by allocated q QCs at time step t in the baseline plan and 0 otherwise; $i \in V, q_i \in \left[0, q_i^{min}, \dots, q_i^{max}\right], t \in T$.
Y_{it}^q : set to 1, if vessel i is served by allocated q QCs at time step t as buffer QCs in the baseline plan and 0 otherwise; $i \in V, q_i \in \left[0, \dots, q_i^{min}\right], t \in T$
q_{it}: number of QCs should be allocated for serving vessel i at time step t; $i \in V, t \in T$.
ρ_t: number of QCs allocated to serve all moored vessels at time step t; $t \in T$.
z_{ikt}: set to 1, if vessel i is served by QC k at time step t and 0 otherwise; $i \in V, k \in Q, t \in T$.
L_{ikt}: set to 1, if the QC assigned to the leftmost position of vessel i at time step t is QC k and 0 otherwise; $i \in V, k \in Q, t \in T$.
R_{ikt}: set to 1, if the QC assigned to the rightmost position of vessel i at time step t is QC k and 0 otherwise; $i \in V, k \in Q, t \in T$.
b_{is}: berthing location of vessel i in scenario s;$i \in V, s \in S$.
Δb_{is}: discrepancy between the expected berthing location, b_i^o, and berthing location of vessel i at berth section b_{is} in scenario s; $i \in V, s \in S$.
α_{is}: the beginning of berthing time of vessel i in scenario s; $i \in V, s \in S$.
β_{is}: the finish of berthing time of vessel i in scenario s; $i \in V, s \in S$..
θ_{its}: if vessel i is being serviced in scenario s at time step t, set to 1, otherwise set to 0.; $i \in V, t \in T, s \in S$.
ω_{ibs}: if the center of the vessel length i is located at berth section b in scenario s, set to 1; otherwise, set to 0; $i \in V, b \in B, s \in S$.
ξ_{ibs}: if vessel i moors at berth section b in scenario s, set to 1; otherwise, set to 0.; $i \in V, b \in B, s \in S$.
π_{its}: set to 1, if vessel i's berthing time in scenario s starts at time step $t = \alpha_{is}$, otherwise set to 0; $i \in V, t \in T |\ t \leq H, s \in S$.
φ_{its}: set to 1, if vessel i's buffer time in scenario s finishes at time step $t \geq \beta_{is}$, otherwise set to 0; $i \in V, t \in T |\ t \leq H, s \in S$.
X_{its}^q: set to 1, if vessel i is served by allocated q QCs at time step t in scenario s, 0 otherwise; $i \in V, q_i \in \left[0, q_i^{min}, \dots, q_i^{max}\right], t \in T, s \in S$.

Y^q_{its}: set to 1, if vessel i is served by allocated q QCs at time step t as buffer QCs in scenario s, 0 otherwise; $i \in V, q_i \in [0, q_i^{min}, \dots, q_i^{max}], t \in T, s \in S$.

q_{its}: number of QCs should be allocated for serving vessel i at time step t in scenario s; $i \in V, t \in T, s \in S$.

ρ_{ts}: number of QCs allocated to serve all moored vessels at time step t in scenario s; $t \in T, s \in S$.

z_{ikts}: set to 1, if vessel i is served by QC k at time step t in scenario s and 0 otherwise; $i \in V, k \in Q, t \in T, s \in S$.

L_{ikts}: set to 1, if the QC assigned to the leftmost position of vessel i at time step t in scenario s is QC k and 0 otherwise; $i \in V, k \in Q, t \in T, s \in S$.

R_{ikts}: set to 1, if the QC assigned to the rightmost position of vessel i at time step t in scenario s is QC k and 0 otherwise; $i \in V, k \in Q, t \in T, s \in S$.

δ^y_{ijs}: set to 1, if vessel i's berthing location along the quay wharf area in scenario s is on the left side of vessel j, and 0 otherwise; $i, j \in V, s \in S$.

δ^x_{ijs}: set to 1, if vessel i's handling time is completed before vessel j's handling time begins in scenario s, 0 otherwise; $i, j \in V, s \in S$.

τ^{a+}_{is}: waiting time for mooring after vessel i's arrival time a_{is} in scenario s; $i \in V, s \in S$.

τ^{b+}_{is}: tardiness of vessel i's mooring finish time in scenario s; $i \in V, s \in S$.

γ_{is}: vessel i's relative tardiness in comparison to the baseline plan in scenario s; $i \in V, s \in S$.

δ_{is}: vessel i's handling time duration exceeds the buffer time in scenario s; $i \in V, s \in S$.

λ_{its}: number of additional QCs setup for a vessel i at time step t as a recovery plan in scenario s compared to the baseline plan; $i \in V, t \in T, s \in S$.

There are two objective functions in the proposed recoverable robust optimization model. The first objective is to maximize the minimum service level of all vessels in the baseline plan as presented in (1).

Max. The minimum service level.

$$Max.\ SL = \min_{i \in V}\{SL_i\} \quad (1)$$

where: $SL_i = 1 - \frac{\tau_i^{a+} + \tau_i^{b+}}{b_i^e - a_i^e}$ $\quad \forall i \in V \quad (2)$

The cost efficiency is expressed as the second objective, which is to minimize the total cost of the baseline plan, the total cost of recovery related to the baseline plan for each scenario, and the total cost expectation of all solutions in the scenarios.

$$Min. TC = TC_b + TC_r + TC_s \quad (3)$$

As indicated in (4), the total costs of the baseline plan include the cost of waiting before mooring, the cost of delayed departure, and the QCs operating costs allocated to all vessels in the baseline plan.

$$TC_b = \sum_{i \in V} c_{1i}.\tau^{a+}_{is} + \sum_{i \in V} c_{1i}.\tau^{b+}_{is} + \sum_{i \in V}\sum_{q \in q_i}\sum_{t \in T} c_2 q X^q_{it} \quad (4)$$

Total recovery costs consist of total cost of postponement due to recovery plan, total cost of operations lateness after end of buffer time, and total cost of QC setup due to recovery plan as stated in (5).

$$TC_r = \sum_{s \in S} p_s\{\sum_{i \in V} c_{1i}(\gamma_{is} + \delta_{is}) + \sum_{i \in V}\sum_{t \in T} c_3 \lambda_{its}\} \quad (5)$$

The expected total cost of all scenarios is comprised of the expected cost of waiting time before mooring costs, the cost of delayed departure, and the QC operation costs, as expressed in (6).

$$TC_s = \sum_{s \in S} p_s \{ \sum_{i \in V} c_{1i}.\tau_{is}^{a+} + \sum_{i \in V} c_{1i}.\tau_{is}^{b+} + \sum_{i \in V} \sum_{q \in q_i} \sum_{t \in T} c_2 q X_{its}^{q} \} \quad (6)$$

Berth allocation related constraints

The formulas (7) – (19) describe the constraints associated with berth allocation for each vessel in the quay wharf area within the planning horizon.

$$\Delta b_i \geq b_i - b_i^o \quad \forall i \in V \quad (7)$$
$$\Delta b_i \geq b_i^o - b_i \quad \forall i \in V \quad (8)$$
$$\sum_{b \in B} \omega_{ib} = 1 \quad \forall i \in V \quad (9)$$
$$\sum_{b \in B} b.\omega_{ib} = b_i \quad \forall i \in V \quad (10)$$
$$s \sum_{b \in B} (b-1)\omega_{ib} \leq b_i \quad \forall i \in V \quad (11)$$
$$b_i \leq s \sum_{b \in B} b.\omega_{ib} - 1 \quad \forall i \in V \quad (12)$$
$$\xi_{i1} = \omega_{i1} \quad \forall i \in V \quad (13)$$
$$\xi_{ib} \geq \omega_{ib} \quad \forall i \in V, b \in B \quad (14)$$
$$\xi_{ib} + \omega_{ib} \leq 1 \quad \forall i \in V, b \in B \quad (15)$$
$$\xi_{ib} \leq \xi_{ib-1} + \omega_{ib} \quad \forall i \in V, b \in B \quad (16)$$
$$s_b \leq \alpha_i + M.(1 - \omega_{ib}) \quad \forall i \in V, \forall b \in B \quad (17)$$
$$e_b \geq \beta_i - M.(1 - \omega_{ib}) \quad \forall i \in V, \forall b \in B \quad (18)$$
$$e_b - s_b \leq H \quad \forall b \in B \quad (19)$$

The discrepancy between expected berthing location, b_i^o, and vessel i's berthing location at berth section b_i is defined by constraints (7) and (8). Constraints (9) and (10) state that the vessel's center location must be in one of the berth sections. If $\omega_{ib} = 1$, the berthing location of vessel i's centre position, b_i, is within berth section b's range. According to constraints (11) and (12), the interval of $[s.(b-1), s.b-1]$ along the quay wharf area is covered by the berth section b. The relationship between ξ_{ib} and ω_{ib} is described by constraints (13) – (16). Constraints (17) and (18) define the berth section b occupied at the start and finish times. According to constraint (19), the duration time of berth section b occupied, $e_b - s_b$, does not exceed the planning horizon H.

QC assignment related constraints

The constraints associated with the quay crane (QC) assignment for serving each vessel within the planning horizon are indicated in formulas (20) – (38).

$$\sum_{t \in T} \sum_{q \in q_i} q^{\alpha} X_{it}^{q} \geq m_i (1 + \Delta b_i \beta) \quad \forall i \in V \quad (20)$$

According to constraint (20), each vessel's QC capacity requirement is determined by the QC's productivity loss due to QC interference, as well as the difference between the berthing location and the expected berthing location. Constraint (20) is non-linear inequalities in q^{α}. Where α is an interference exponent, with the value of $0 < \alpha \leq 1$. This given parameter is used to describe the QC's productivity loss. Perceptibly, $q_i \in \{0, q_i^{min}, \cdots, q_i^{max}\}$, which has $q_i^{max} - q_i^{min} + 2$ possible values. Constraint (20) can be substituted by Constraints (21) - (23) to linearize it. Here, $X_{it}^{0}, X_{it}^{q_i^{min}}, \cdots, X_{it}^{q_i^{max}}$ are 0–1 binary variables.

$$X_{it}^{0} + X_{it}^{q_i^{min}} + \cdots + X_{it}^{q_i^{max}} = 1 \quad \forall i \in V, t \in T |\ t \leq H \quad (21)$$

$$q_{it} = 0.X_{it}^{0} + q_i^{min}.X_{it}^{q_i^{min}} + \cdots + q_i^{max}.X_{it}^{q_i^{max}} \quad \forall i \in V, t \in T |\ t \leq H \quad (22)$$

$$\sum_{t=1}^{H}\left[0X_{it}^{0}+\left(q_i^{min}\right)^{\alpha}X_{it}^{q_i^{min}}+\cdots+(q_i^{max})^{\alpha}X_{it}^{q_i^{max}}\right]\geq m_i(1+\Delta b_i\beta) \qquad \forall i\in V \qquad (23)$$

Constraint (21) ensures that X_{it}^{q} must be one of the set members in $\{0, q_i^{min}, \cdots, q_i^{max}\}$. Constraint (22) relates the number of allocated QCs for serving vessel i in time step t, q_{it} and $X_{it}^{0}, X_{it}^{q_i^{min}}, \cdots, X_{it}^{q_i^{max}}$. Constraint (23) guarantees the fulfilment of the QC capacity demand of vessel i.

$$\sum_{\substack{q\in q_i\\ q\neq 0}} X_{it}^{q}=\theta_{it} \qquad \forall i\in V, t\in T \qquad (24)$$
$$\sum_{t\in T}\theta_{it}=\beta_i-\alpha_i+1 \qquad \forall\, i\in V \qquad (25)$$
$$t.\theta_{it}\leq\beta_i \qquad \forall i\in V, \forall t\in T \qquad (26)$$
$$t.\theta_{it}+H(1-\theta_{it})\geq\alpha_i \qquad \forall i\in V, \forall t\in T \qquad (27)$$
$$\alpha_i\geq a_i^{f} \qquad \forall\, i\in V \qquad (28)$$
$$\alpha_i-a_i^{e}=\tau_i^{a+}-\tau_i^{a-} \qquad \forall i\in V \qquad (29)$$
$$\beta_i-\beta_i^{e}=\tau_i^{b+}-\tau_i^{b-} \qquad \forall i\in V \qquad (30)$$
$$\tau_i^{a+}\geq\alpha_i-a_i^{e} \qquad \forall i\in V \qquad (31)$$
$$\tau_i^{b+}\geq\beta_i-b_i^{e} \qquad \forall i\in V \qquad (32)$$
$$\sum_{t\in T}\varphi_{it}=1 \qquad \forall i\in V \qquad (33)$$
$$\beta_i-t\leq M(1-\varphi_{it}) \qquad \forall i\in V, t\in T \qquad (34)$$
$$\alpha_j+M\left(1-\delta_{ij}^{x}\right)\geq\sum_{t\in T}\varphi_{it} \qquad \forall\, i,j\in V, i\neq j \qquad (35)$$
$$b_j-\frac{1}{2}l_j+M\left(1-\delta_{ij}^{y}\right)\geq b_i+\frac{1}{2}l_i \qquad \forall i,j\in V, i\neq j \qquad (36)$$
$$\delta_{ij}^{x}+\delta_{ji}^{x}+\delta_{ij}^{y}+\delta_{ji}^{y}\geq 1 \qquad \forall i,j\in V, i\neq j \qquad (37)$$
$$\delta_{ji}^{y}+\omega_{ib}+\sum_{b'=max[b-l_i+1,1]}^{L}\omega_{ib'}\leq 2 \qquad \forall i,j\in V, i\neq j, b\in B \qquad (38)$$

Constraint (24) indicates the relationship between the number of allocated QCs and the vessel's berthing time. The vessel should moor at the quay wharf area if there are q QCs designated to serve vessel i in time step t. Constraint (25) guarantees that the vessel's berthing time is equal to the number of time steps during which QCs operate to serve the vessel. Constraints (26) and (27) imply that the QCs operations will not be preempted, and the start and finish times of the vessel berthing will be appropriately determined. Constraint (28) ensures that the vessel's berthing begins after the vessel's feasible arrival time. Constraint (29) connects $\alpha_i-a_i^{e}$ and $\tau_i^{a+}-\tau_i^{a-}$. The relationship between $\beta_i-\beta_i^{e}$ and $\tau_i^{b+}-\tau_i^{b-}$ is shown in constraint (30). The waiting time before vessel berthing, τ_i^{a+}, and the tardiness time from the expected departure time, τ_i^{b+}, are defined by constraints (31) and (32). Constraint (33) indicates that there is only one time step to end of the buffer time for each vessel. Constraint (34) correctly determines the ending of the buffer times. The ending of the buffer time (φ_{it}) must be greater than or equal to the end of the QC operating time (β_i). The values of variables δ_{ij}^{y} and δ_{ij}^{x} are set using constraints (35) and (36) respectively. By considering their time windows, baseline plan designs should not overlap in space along the quay wharf. The vessel i's berthing time window is from $\alpha_i$to φ_{it}. If $\delta_{ij}^{x}=1$, for any two vessels, i and j, then vessel i's rightmost berthing location is less than or equal to vessel j's leftmost berthing location. Constraint (37) applies these variables to prevent overlaps in the berth plan diagram. If vessel i's berthing location is on the left side of vessel j, constraint (38) assures that vessel j can only moor on the right side of vessel i's berthing location.

Berth - QC related constraints

The formulas (39) – (61) demonstrate the constraints associated with the integration of berth allocation and quay crane assignment to serve each vessel over the planning horizon.

$\sum_{q\in\{1,\cdots,q_i^{min}\}} Y_{it}^q \le 1 \qquad \forall i \in V, t \in T \qquad (39)$

$\sum_{q\in q_i} qX_{it}^q + \sum_{q\in\{1,\cdots,q_i^{min}\}} qY_{it}^q \le q_i^{max} \qquad \forall i \in V, t \in T \qquad (40)$

$\sum_{q\in\{1,\cdots,q_i^{min}\}} \sum_{t'\in\{1,\cdots,t-1\}} Y_{it'}^q \le (1-\pi_{it})H \qquad \forall i \in V, t \in T \qquad (41)$

$\sum_{q\in\{1,\cdots,q_i^{min}\}} \sum_{t'\in\{t,\cdots,H\}} Y_{it'}^q \le (1-\varphi_{it})H \qquad \forall i \in V, t \in T \qquad (42)$

$\sum_{i\in V} \sum_{q\in q_i} qX_{it}^q + \sum_{i\in V} \sum_{q\in\{1,\cdots,q_i^{min}\}} qY_{it}^q \le Q \qquad \forall t \in T \qquad (43)$

$\sum_{t\in T} t.\pi_{it} = \alpha_i \qquad \forall i \in V \qquad (44)$

$\sum_{t\in T} \pi_{it} = 1 \qquad \forall i \in V \qquad (45)$

$\pi_{it} = 0 \qquad \forall i \in V, t \in \{T | t < a_i^f\} \qquad (46)$

$\theta_{i1} = \pi_{i1} \qquad \forall i \in V \qquad (47)$

$\theta_{it} \le \theta_{it-1} + \pi_{it} \qquad \forall i \in V, t \in T \qquad (48)$

$\sum_{t\in T} \vartheta_{it} t = \beta_i \qquad \forall i \in V \qquad (49)$

$\sum_{t\in T} \vartheta_{it} = 1 \qquad \forall i \in V \qquad (50)$

$\vartheta_{it} = 0 \qquad \forall i \in V, t \in \{T | t > b_i^f\} \qquad (51)$

$\theta_{iH-1} = \vartheta_{iH} \qquad \forall i \in V \qquad (52)$

$\theta_{it-1} \le \theta_{it} + \vartheta_{it} \qquad \forall i \in V, t \in T \qquad (53)$

$\sum_{i\in V} z_{ikt} \le 1 \qquad \forall k \in Q, t \in T \qquad (54)$

$z_{ikt} - z_{ik-1t} \ge 1 - M(1 - L_{ikt}) \qquad \forall i \in V, \forall k \in Q, t \in T |\ t \le H \qquad (55)$

$z_{ikt} - z_{ik-1t} \le ML_{ikt} \qquad \forall i \in V, \forall k \in Q, t \in T |\ t \le H \qquad (56)$

$z_{ikt} - z_{ik+1t} \ge 1 - M(1 - R_{ikt}) \qquad \forall i \in V, \forall k \in Q, t \in T |\ t \le H \qquad (57)$

$z_{ikt} - z_{ik+1t} \le M.R_{ikt} \qquad \forall i \in V, \forall k \in Q, t \in T |\ t \le H \qquad (58)$

$\sum_{k\in Q} L_{ikt} \le \theta_{it} \qquad \forall i \in V, t \in T |\ t \le H \qquad (59)$

$\sum_{k\in Q} R_{ikt} \le \theta_{it} \qquad \forall i \in V, t \in T |\ t \le H \qquad (60)$

$\sum_{k\in Q} z_{ikt} = q_{it} \qquad \forall i \in V, t \in T |\ t \le H \qquad (61)$

The number of buffer QCs allocated to each vessel is determined by constraints (39)–(42). According to constraint (39), the number of QCs allocated to each vessel for each time step must be between 1 to r_i^{max}. Constraint (40) assures that the total number of QCs allocated to each vessel, including the buffer QCs, does not exceed q_i^{max} at any time step. The buffer QCs for vessel i can only be allocated between time stepsα_i and φ_{it}, as stated in constraints (41) and (42). The total number of QCs allocated must not exceed the number of QCs available at the quay wharf area, according to constraint (43). Constraint (44), which depicts the relationship between variables a and b, is important for removing the region of infeasible solutions. Constraint (45) ensures that each vessel can only begin berthing at a specific time step, while constraint (46) guarantees that the start berthing time cannot be earlier than the vessel's arrival time. Constraints (47) - (49) explain that if the vessel is in the quay wharf area, it may begin berthing at the time step or be berthing at the prior time step. The finish berthing time is constrained in a similar way. If vessel i's berthing time finishes at time step t, the auxiliary binary variable, ϑit, is set to 1, otherwise it is set to 0. Constraints (44)-(48) define the beginning of berthing time for each vessel, and constraints (49)-(53) describe the end of berthing time for each vessel in the same way. If a vessel is at berth in time step $t-1$, then either the following time step is the ending time step or it is still at berth in the next time step t, according to constraint (53). Constraint (54) indicates that each QC can only serve one vessel at a time step. The relationship between variables L_{ikt} and R_{ikt} is defined by constraints (55)- (58).

Constraints (59)– (61) represent the assignment of QCs to the leftmost, rightmost, and in-between positions.

Recovery plan related constraints

The constraints (62) – (99) are used to build a recovery plan for each scenario based on the application of the previous constraints' baseline plan.

$$\sum_{t\in T}\sum_{q\in q_i} q^{\alpha} X_{its}^{q} \geq m_{is}(1+\Delta b_{is}.\beta) \qquad \forall i\in V, t\in T, s\in S \qquad (62)$$

$$X_{its}^{0} + X_{its}^{q_i^{min}} + \cdots + X_{its}^{q_i^{max}} = 1 \qquad \forall i\in V,\ t\in T|\ t\leq H, s\in S \qquad (63)$$

$$q_{its} = 0.X_{its}^{0} + q_i^{min}.X_{its}^{q_i^{min}} + \cdots + q_i^{max}.X_{its}^{q_i^{max}} \qquad \forall\ i\in V, t\in T|\ t\leq H,\ s\in S \qquad (64)$$

$$\sum_{t=1}^{H}\left[0X_{its}^{0} + \left(q_i^{min}\right)^{\alpha} X_{its}^{q_i^{min}} + \cdots + (q_i^{max})^{\alpha} X_{its}^{q_i^{max}}\right] \geq m_{is}(1+\Delta b_{is}.\beta) \qquad \forall i\in V \qquad (65)$$

$$\sum_{q\in q_i} X_{its}^{q} = \theta_{its} \qquad \forall\ i\in V, t\in T, s\in S \qquad (66)$$

$$\sum_{t\in T}\theta_{its} = \beta_{is} - \alpha_{is} \qquad \forall\ i\in V, s\in S \qquad (67)$$

$$t.\theta_{its} \leq \beta_{is} \qquad \forall\ i\in V, t\in T, s\in S \qquad (68)$$

$$t.\theta_{its} + H(1-\theta_{its}) \geq \alpha_{is} \qquad \forall\ i\in V, t\in T, s\in S \qquad (69)$$

$$\alpha_{is} \geq a_i^{e} \qquad \forall i\in V, s\in S \qquad (70)$$

$$\Delta b_{is} \geq b_{is} - b_i^{o} \qquad \forall\ i\in V, s\in S \qquad (71)$$

$$\Delta b_{is} \geq b_i^{o} - b_{is} \qquad \forall\ i\in V, s\in S \qquad (72)$$

$$b_{js} - \frac{1}{2}l_j + M\left(1-\delta_{ijs}^{y}\right) \geq b_{is} + \frac{1}{2}l_i \qquad \forall\ i,j\in V, i\neq j, s\in S \qquad (73)$$

$$\alpha_{js} + M\left(1-\delta_{ijs}^{x}\right) \geq \beta_{is} \qquad \forall i,j\in V, i\neq j, s\in S \qquad (74)$$

$$\delta_{ijs}^{x} + \delta_{jis}^{x} + \delta_{ijs}^{y} + \delta_{jis}^{y} \geq 1 \qquad \forall\ i,j\in V, i\neq j, s\in S \qquad (75)$$

$$\sum_{b\in B} b.\omega_{ibs} = b_{is} \qquad \forall\ i\in V, s\in S \qquad (76)$$

$$\sum_{b\in B}\omega_{ibs} = 1 \qquad \forall i\in V, s\in S \qquad (77)$$

$$\xi_{i1s} = \omega_{i1s} \qquad \forall i\in V, s\in S \qquad (78)$$

$$\xi_{ibs} \geq \ \omega_{ibs} \qquad \forall i\in V, b\in B, s\in S \qquad (79)$$

$$\xi_{ibs} + \omega_{ibs} \leq 1 \qquad \forall i\in V, b\in B, s\in S \qquad (80)$$

$$\xi_{ibs} \leq \xi_{ib-1s} + \omega_{ibs} \qquad \forall i\in V, b\in B, s\in S \qquad (81)$$

$$\delta_{jis}^{y} +\ \omega_{ibs} + \sum_{b'=max[b-l_i+1,1]}^{L}\ \omega_{ib's} \leq 2 \qquad \forall i,j\in V, i\neq j, b\in B, s\in S \qquad (82)$$

$$s_{bs} \leq \alpha_{is} + M.(1-\omega_{ibs}) \qquad \forall i\in V, \forall b\in B, s\in S \qquad (83)$$

$$e_{bs} \geq \beta_{is} - M.(1-\omega_{ibs}) \qquad \forall i\in V, \forall b\in B, s\in S \qquad (84)$$

$$e_{bs} - s_{bs} \leq H \qquad \forall b\in B, s\in S \qquad (85)$$

$$\sum_{i\in V}\sum_{q\in q_i} q\, X_{its}^{q} \leq Q \qquad \forall t\in T, s\in S \qquad (86)$$

$$\sum_{i\in V} z_{ikts} \leq 1 \qquad \forall k\in Q, t\in T, s\in S \qquad (87)$$

$$z_{ikts} - z_{ik-1ts} \geq 1 - M(1-L_{ikts}) \qquad \forall i\in V, \forall q\in Q, t\in T|\ t\leq H, s\in S \qquad (88)$$

$$z_{ikts} - z_{ik-1ts} \leq ML_{ikts} \qquad \forall i\in V, \forall k\in Q, t\in T|\ t\leq H, s\in S \qquad (89)$$

$$z_{ikts} - z_{ik+1ts} \geq 1 - M(1-R_{ikts}) \qquad \forall i\in V, \forall k\in Q, t\in T|\ t\leq H, s\in S \qquad (90)$$

$$z_{ikts} - z_{ik+1ts} \leq M.R_{ikts} \qquad \forall i\in V, \forall k\in Q, t\in T|\ t\leq H, s\in S \qquad (91)$$

$$\sum_{k\in Q} L_{ikts} \leq \theta_{its} \qquad \forall i\in V, t\in T|\ t\leq H, s\in S \qquad (92)$$

$$\sum_{k\in Q} R_{ikt} \leq \theta_{its} \qquad \forall i\in V, t\in T|\ t\leq H, s\in S \qquad (93)$$

$$\sum_{k\in Q} z_{ikts} = q_{its} \qquad \forall i\in V, t\in T|\ t\leq H, s\in S \qquad (94)$$

$$\gamma_{is} \geq \beta_{is} - \beta_i\ - (\alpha_{is} - a_i^{e}) \qquad \forall t\in T, s\in S \qquad (95)$$

$$\delta_{is} \geq \beta_{is} - \sum_{t\in T} t.\varphi_{it} \qquad \forall i\in V, s\in S \qquad (96)$$

$$\lambda_{its} + \sum_{q\in\{1,\cdots,r_i^{min}\}} qY_{it}^{q} \geq \sum_{q\in q_i} qX_{its}^{q} - \sum_{q\in q_i} qX_{it}^{q} \qquad \forall i\in V, t\in T, s\in S \qquad (97)$$

$$\tau_{is}^{a+} \geq \alpha_{is} - a_i^{e} \qquad \forall i\in V, s\in S \qquad (98)$$

$$\tau_{is}^{b+} \geq \beta_{is} - b_i^e \qquad \forall i \in V, s \in S \qquad (99)$$
$$\tau_i^{a+}, \tau_i^{b+}, \alpha_i, \beta_i \in \{1, \cdots, H\} \qquad \forall i \in V \qquad (100)$$
$$b_i, \Delta b_i \in \{1, \cdots, L - l_i\} \qquad \forall i \in V \qquad (101)$$
$$\varphi_{it}, \theta_{it}, \pi_{it} \in \{0,1\} \qquad \forall i \in V, t \in T \qquad (102)$$
$$X_{it}^q, Y_{it}^q, \delta_{ij}^x, \delta_{ij}^y \in \{0,1\} \qquad \forall i, j \in V, i \neq j, t \in T, q \in q_i \qquad (103)$$
$$\gamma_{is}, \delta_{is}, \tau_{is}^{a+}, \tau_{is}^{b+} \in \{1, \cdots, H\} \qquad \forall i \in V, t \in T, s \in S \qquad (104)$$
$$\lambda_{its} \in \{0, \cdots, Q\} \qquad \forall i \in V, t \in T, s \in S \qquad (105)$$
$$b_{is}, \Delta b_{is} \in \{1, \cdots, L - l_i\} \qquad \forall i \in V, s \in S \qquad (106)$$
$$X_{its}^q, Y_{its}^q, \delta_{ijs}^x, \delta_{ijs}^y \in \{0,1\} \qquad \forall i, j \in V, i \neq j, t \in T, q \in q_i, s \in S \qquad (107)$$

Constraints (62) - (94) have the same meaning as the constraints (7) - (61) that apply to each scenario *s*. The recovery plans for each scenario are determined by these constraints. The recovery plan's performance is compared to the baseline plan's results in Constraints (95) - (97). Constraint (95) establishes the γ_{is} variables' values, as well as the vessel *i's* tardiness time in scenario *s* based on the baseline plan's performance. Constraint (96) states δ_{is} variable which is the duration of the vessel *i's* tardiness time in scenario *s* concerning the end of the buffer time. The value of this variable indicates the requirement of rescheduling efforts in the storage container yard and quay wharf area. Constraint (97) calculates the number of additional QC setups for vessel *i* in scenario s. QCs setup for a vessel *i* in time step *t* are required only if the recovery plan for scenario s requires more QCs than the number of QCs allocated in the baseline plan. Constraints (98) and (99) indicate the waiting time before mooring and the tardiness of completion time for each scenario solution. Furthermore, the start berthing time, finish berthing time, waiting time, operations lateness time, and berthing location decision variable domains are described in constraints (100) and (101). Finally, constraints (102) and (103) address the binary variables, while all variables' domains in each scenario are defined by constraints (104) through (107).