International Journal of Technology (2018) 5: 1015-1026
ISSN 2086-9614

MEASUREMENT AND PREDICTION OF THE DENSITY AND VISCOSITY OF BIODIESEL BLENDS

Minh Tuan Pham[1*], Anh Tuan Hoang[2], Anh Tuan Le[1], Abdel Rahman M.Said Al-Tawaha[3], Van Huong Dong[2], Van Vang Le[2]

[1]*Hanoi University of Science and Technology, Hanoi, Vietnam*
[2]*Ho Chi Minh city University of Transport, Ho Chi Minh, Vietnam*
[3]*Al-Hussein Bin Talal University, Maan, Jordan*

(Received: April 2018 / Revised: July 2018 / Accepted: October 2018)

ABSTRACT

Biodiesel has been considered as the potential fuel type with many advantages such as environmental pollution reduction, no sulfur production, and biodegradation. However, disadvantages of biodiesel such as high viscosity and high density affected diesel engines and fuel systems negatively. Thus, it is necessary to reduce the viscosity and density of biodiesel fuel in unmodified diesel engines. Until now, a large number of empirical correlations have been used to predict the viscosity and density of biodiesel–fossil diesel fuel blend This study was conducted to predict the kinematic viscosity and density of blends of biodiesel and fossil diesel fuel. Three types of biodiesel were examined: Coconut oil-based biodiesel (COB), Jatropha oil-based biodiesel (JOB), and Waste oil-based biodiesel (WOB). Twenty-four samples of the three types of biodiesel–diesel fuel blends were created by blending 5% (B5), 10% (B10), 20% (B20), 40% (B40), 50% (B50), 60% (B60), 75% (B75), and 100% (B100) of biodiesel with conventional diesel fuel to produce the corresponding blends for experimental purposes. Experimental correlations and mathematical equations for predicting the relationship between the kinematic viscosity and the density of the biodiesel–fossil diesel fuel blends, the dependence of the kinematic viscosity and the density of the biodiesel–fossil diesel fuel blends on biodiesel fractions, and the effects of temperature on the kinematic viscosity and density of pure biodiesel were developed. The results of the experimental correlation data were near the predicted mathematical equation with a confidence level of 95%.

Keywords: Biodiesel; Biodiesel fraction; Density; Kinematic viscosity; Temperature

1. INTRODUCTION

The issues related to the increase in energy demand, fossil fuel depletion, and environmental pollution are considered so urgent that we must find renewable and alternative fuels to replace fossil fuels with the aim of maintaining fresh air and ensuring energy safety (Abubakar et al., 2016; Anh Tuan & Minh Tuan, 2009). Biodiesel produced by the transesterification reaction of oils or fats that are originated from edible and non-edible vegetable oils and animal fats are considered potential alternative fuels with environmentally friendly, non-sulfur, and non-toxic properties (Rajagopal et al., 2016; Liu et al., 2017). In particular, biodiesel is prone to blending with mineral diesel fuels to form homogeneous blends, which are used in diesel engines without any modifications (Hoang & Le, 2017). However, the viscosity and density of biodiesel are

---

*Corresponding author's email: tuan.phamminh@hust.edu.vn, Tel. +84-912094727, Fax. +84-24-38684945
Permalink/DOI: https://doi.org/10.14716/ijtech.v9i5.1950

higher than those of fossil diesel fuels because of the higher molecular mass of biodiesel compared with that of fossil diesel fuel (Leong et al., 2016; Hoang & Nguyen, 2017). High viscosity and density lead to poor atomization, time-consuming breakup and mixture formation, increased carbon deposition, and a high energy demand for pumping fuel (Aliyah et al., 2016). Until now, some techniques, such as blending, preheating, and emulsification, have been applied to improve the above-mentioned disadvantages of biodiesel (Suryantoro et al., 2016; Hoang et al., 2018).

Large empirical correlations between biodiesel properties (including viscosity and density) and temperature or fraction have been developed. The influence on viscosity and density of blending biodiesel and traditional fossil diesel fuels was studied by scientists (Tran et al., 2017), and empirical and mathematical equations were proposed to allow the calculation of the blends of biodiesel and diesel fuel viscosity and density (Saxena et al., 2013; Hoang & Pham, 2018). These equations can be used to predict the viscosity and density of blends of biodiesel and diesel fuel through several parameters, such as the molecular mass of biodiesel, temperature, and volume percentage (Chavarria-Hernandez & Pacheco-Catalán, 2014). As reported, the specific density (ρ) of fuel is calculated as the ratio of fuel mass and fuel volume based on the same condition. Gülüm and Bilgin (2017) suggested the 1st degree linear shown in Equation 1 for the correlation of density (ρ) and temperature (T). This relationship was established when pure soybean biodiesel (SOB100) and its blends, such as B20 (20% of SOB), B50 (50% of SOB), and B75 (75% of SOB), were used experimentally to measure density based on the hydrometer method. However, Yang et al. (2018) proposed that biodiesel density should be considered as a function of the mass fractions of components; thus, it could be calculated as shown in Equation 2. In addition, Bhale et al. (2009) determined the density of a biodiesel blend by using Equation 3. From Equation 3, it showed the relationship between the density of the biodiesel blend and the mass or volume percentage of the components:

$$\rho_{biodiesel} = a + bT \tag{1}$$

$$\rho_{biodiesel} = \sum_{i=1}^{n} \rho_i M_i \tag{2}$$

$$\rho_{biodiesel} = A + Bx \tag{3}$$

The viscous property of a fluid characterizes the resistance to the liquid flow, and it is inversely proportional to the flow velocity. High viscosity is thought to be the main cause of sedimentation in equipment (Hoang, 2017). The high viscosity of the fuel used in diesel engines leads to poor atomization and difficulty in evaporation (Hoang & Le, 2018). Normally, the kinematic viscosity of biodiesel is 2–4 times higher than that of conventional diesel fuel at room temperature (Tat & Van Gerpen, 2000). The reduction of the kinematic viscosity of biodiesel can be achieved through the preheating method, blending with low-viscosity fuel, and using additives. Albuquerque et al. (2009) used the Grun-Nissan and Katti-Chaudhri laws to estimate the kinematic viscosity of biodiesel. The mathematical formula shown in Equation 4 was derived from the results of their study. Riazi and Al-Otaibi (Camas-Anzueto et al., 2017) proposed another equation (Equation 5) for estimating and predicting the dynamic viscosity of fluid fuels that include hydrocarbons. However, this equation has been little used because of the difficulty in determining the input molecular weight as well as the refractive index of compounds. Regarding cases where the dependence of biodiesel viscosity on temperature is considered, Equation 6 was developed by Fasina and Colley (2008) to determine the kinematic viscosity of biodiesel the temperature range from 5^{o}C to 300^{o}C with an absolute error of 22.3% and the correlation coefficient $R^2 \approx 100\%$.

$$\ln KV = x_1 \ln KV_1 + x_2 \ln KV_2 + ... + x_n \ln KV_n \tag{4}$$

$$\frac{1}{\mu} = A + \frac{B}{I} \tag{5}$$

$$\ln KV = -2.4343 + \frac{216.66}{T} + \frac{293523}{T^2} \tag{6}$$

where KV_i - kinematic viscosity of component i, mm^2/s; ρ_i- “i” component specific density, g/cm^3; M_i- “i” component mass fraction, g; x_i- the mass or volume fractions of component i, %; I - the refractive index; T - temperature, K; x - biodiesel fraction, %; a, b, A, B, C- coefficients.

The empirical correlations between the density and kinematic viscosity of biodiesel and its blends and the percentages of the mass and volume of the components or the temperature can be determined by the above-mentioned equations (from Equation 1 to Equation 6). Furthermore, the density and kinematic viscosity of each type of fuel are considered two important parameters, which were used to evaluate its applicability to diesel engines, the simulation process, combustion modeling, and the prediction of exhaust emission. Thus, the estimation and prediction method can be used for determining the dependence of the kinematic viscosity (KV) and density (ρ) of biodiesel on biodiesel fraction and temperature. The aim of this study is to determine the correlation between biodiesel KV and ρ with temperature and fraction based on the empirical parameters.

2. MATERIALS AND METHODS

2.1. Materials

In this study, three types of biodiesel were used: coconut oil-based biodiesel (COB), *Jatropha* oil-based biodiesel (JOB), and waste oil-based biodiesel (WOB). All biodiesels that were available in Vietnam were used in the analysis. COB, JOB, and WOB were produced only with the pilot scale. COB, JOB, and WOB were products of the transesterification reaction between the above-mentioned vegetable oils with methanol and additives. Fossil diesel fuel was provided by the Vietnam National Petroleum Group (Petrolimex). The properties of COB, JOB, and WOB and the as-used diesel fuel (DF) are shown in Table 1.

Table 1 Physical properties of COB, JOB, WOB, and DF at 30°C

Properties	Unit	COB	JOB	WOB	DF
Density, ρ	kg/m^3	878	880	883	852
Kinematic viscosity, KV	cSt	6.2	6.5	6.8	3.5
Cetane number, CN	-	70	50	52	45
Flash point, F	°C	140	166	180	55
Calorific value, HV	MJ/kg	40.5	41	39.5	43

Based on the volume percentage of biodiesel, 5%, 10%, 20%, 40%, 50%, 60%, 75% and 100% of each biodiesel (COB, JOB, and WOB) were mixed with DF in the same ambient condition to measure biodiesel KV and ρ. Samples of 100% biodiesel, such as COB100, JOB100, and WOB100, were used to evaluate the effects of the preheating temperature on biodiesel KV and biodiesel ρ. In total, 24 biodiesel samples of COB, JOB, and WOB were used to measure KV and ρ.

2.2. Methods

After The ASTM (American Society for Testing and Materials) standards for the measurement of the properties of biodiesel blends are shown in Table 2.

Table 2 ASTM standards for the measurement of biodiesel properties

Properties	Unit	ASTM	Requirement
Density, ρ	kg/m^3	ASTM D1298	860–900
Kinematic viscosity, KV	cSt	ASTM D 445	min 1.9
Cetane number, CN	-	ASTM D 613	min 47
Flash point, F	°C	ASTM D 93	min 130
Calorific value, HV	MJ/kg	ASTM D2015-85	min 40

As shown in Table 2, the ASTM D1298 and ASTM D 445 standard procedures were used to determine the correlation of both density and kinematic viscosity with temperature, and the correlation of both density and kinematic viscosity with fraction. For the density measurement, a glass hydrometer based on an accuracy of three decimal places was used in the range of 0.7–1.0 g/cm^3. In this measurement, a one hundred milliliter cylinder containing the biodiesel blend samples were placed in a bath where the temperature was controlled in the range from 30°C to 100°C to evaluate and collect the temperature-dependent data. For the kinematic viscosity measurement, a Cannon Viscometer tube was used to determine the viscosity of the biodiesel blends in the range of 2–30cSt with the viscometer constant at k = 0.0359. Under room conditions, the time required for a known volume of biodiesel sample to flow through a calibrated glass capillary viscometer tube was measured to determine the kinematic viscosity of the biodiesel samples. A biodiesel bath with a temperature range of 30^{o}C–100^{o}C was also used to measure the influence of temperature on kinematic viscosity. However, the ASTM D 445 standard required the experimental tests to be repeated two times with an accuracy lower than 0.02 cSt. The obtained average value of the twice repeated measurement was considered representative and satisfactorily accurate.

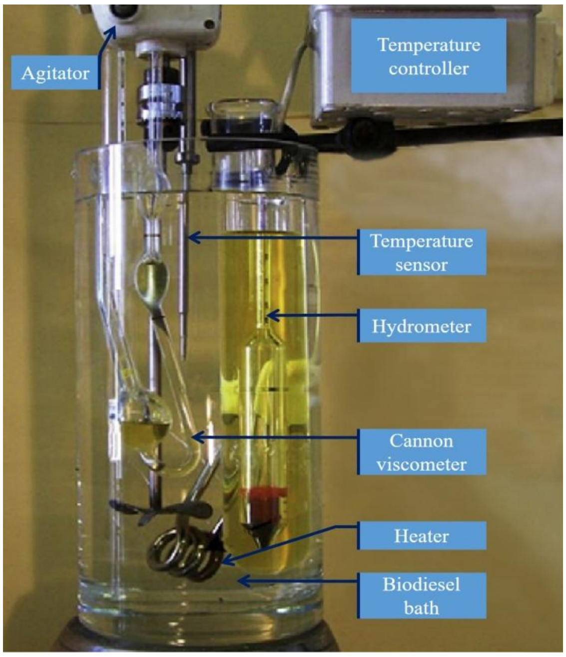

Figure 1 The measurement of biodiesel density and kinematic viscosity

3. RESULTS AND DISCUSSION

3.1. The Relationship between Biodiesel Density, Fraction, and Temperature

Fuel density is a main factor affecting the combustion process, which is comprised of "atomization → breakup → mixture" when fuel is pumped into a combustion chamber. According to Yang et al. (2016), in order to optimize the combustion process, the temperature and fraction dependence of fuel density must be clearly defined. In Equation 1 and Equation 3, biodiesel density was calculated using two parameters: temperature and fraction. The density values of the blends in three different types of biodiesel, ρ_1 (calculated by the ratio of mass and volume), ρ_2 (calculated by Equation 2), and ρ_3 (measured by hydrometer), are shown in Table 3. Δ_1 and Δ_2 are absolute error percentages between ρ_1 and ρ_3 and ρ_1 and ρ_2, respectively. They are calculated as the difference between the evaluated object (A-B) and the object that needs to be compared (B), absolute error = 100%(A-B)/B.

Table 3 Measurement and calculation methods of biodiesel blend density at 30°C

Biodiesel types	Biodiesel fraction (%)	ρ_1, kg/m^3	ρ_2, kg/m^3	ρ_3, kg/m^3	Δ_1	Δ_2
COB	5	851.76	851.36	852.00	0.0470	0.0751
	10	853.41	852.22	853.00	0.1394	0.0914
	20	854.38	853.14	855.00	0.1451	0.2175
	40	856.12	854.47	859.00	0.1927	0.5274
	50	858.48	857.54	861.00	0.1095	0.4019
	60	862.34	861.78	864.00	0.0649	0.2569
	75	867.25	866.35	868.00	0.1038	0.1901
	100	876.30	875.87	877.00	0.0491	0.1288
JOB	5	853.66	852.12	854.00	0.1804	0.2201
	10	855.71	854.08	855.00	0.1905	0.1076
	20	856.58	855.23	857.00	0.1576	0.2065
	40	858.47	856.85	861.00	0.1887	0.4820
	50	859.92	858.19	863.00	0.2012	0.5574
	60	863.73	862.55	866.00	0.1366	0.3984
	75	870.28	868.87	871.00	0.1620	0.2445
	100	878.77	878.44	879.00	0.0376	0.0637
WOB	5	856.53	854.18	857.00	0.2744	0.3291
	10	857.46	856.32	858.00	0.1330	0.1958
	20	859.68	858.21	860.00	0.1710	0.2081
	40	862.45	860.66	864.00	0.2075	0.3866
	50	864.51	863.27	867.00	0.1434	0.4302
	60	867.32	865.81	869.00	0.1741	0.3671
	75	871.65	869.20	874.00	0.2811	0.5492
	100	881.49	880.08	882.00	0.1600	0.2177

As shown in Table 3, the values of the density of the biodiesel blends were proportional to the volume fraction of each type of biodiesel. The higher the biodiesel blend was, the higher the volume fraction. Maximal Δ_1 was calculated to be 0.2811%, and maximal Δ_2 was 0.5574%. In addition, the density values of the 24 samples collected at each random point referring to the same volume fraction were relatively close to each other. This correlation equation is shown in Figure 2 and Equation 7 where *x* is the biodiesel fraction.

$$\rho_{\text{biodiesel}} = 0.2558x + 852.08 \tag{7}$$

As shown in Table 3 and Figure 2, the regression correlation of the experimental values is denoted R^2 and has a value of 0.9844. This result shows that the density correlation between the diesel and the biodiesel was significant. Confidence methods were utilized to predict and determine the density of the biodiesel–diesel blends with different amounts of biodiesel fraction. Equation 7 was used to calculate the hydrometer-based measured values. The results showed that the maximum absolute percentage differences were 0.4956%, 0.2012%, and 0.4463% in WOB, JOB, and COB, respectively.

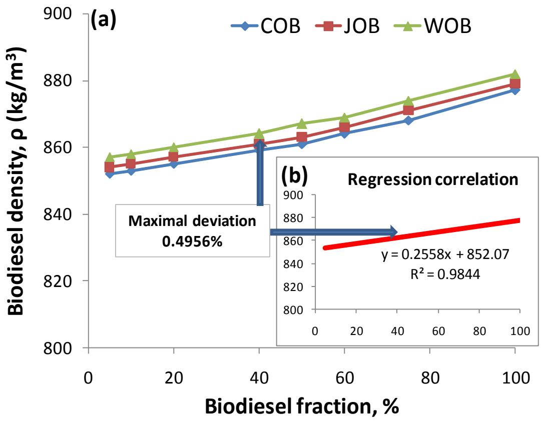

Figure 2 The dependence of biodiesel density on fraction of three studied biodiesel types (a), the regression correlation of experimental values (b)

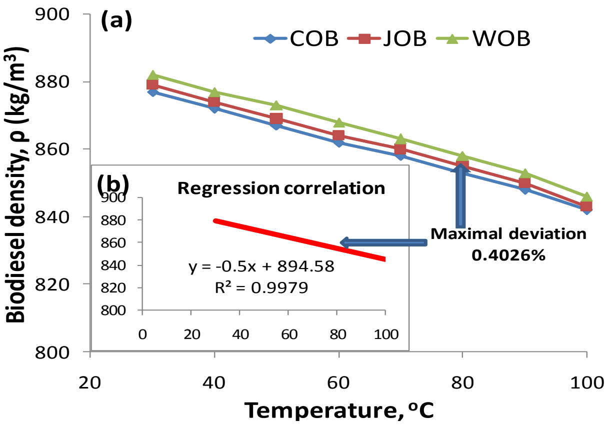

Figure 3 The dependence of biodiesel density on temperature (a), the regression correlation of experimental values (b)

In the second experiment, in order to obtain accurate values of the biodiesel density, the density readings of three uncontaminated biodiesel types were measured at temperatures ranging from 30°C–100°C. As shown in Table 2, the results demonstrated that the biodiesel density was higher than that of the studied DF. According to the European standard (CEN, 2008), the acceptable biodiesel density value is equal to or smaller than 850 kg/m^3. Based on Figure 3, the increase in heating temperature was considered as a method to reduce the density of biodiesel. The dependence of biodiesel density (ρ) on heating temperature (T) was shown in Equation 8.

$$\rho_{biodiesel} = -0.5T + 894.58 \tag{8}$$

Figure 3 shows the results of Equation 8, which yielded the regression correlation of experimental values between biodiesel density and temperature. The value of density was inversely proportional to temperature: R^2 = 0.9979. This result supports previous findings (Tesfa et al., 2010; Esteban et al., 2012). It was also determined that the value of the experimental density range was 878 kg/m^3–883 kg/m^3. The analytical results showed that the heating method was the simplest in reducing the biodiesel density because the cetane number and the heating value did not require adjustment. Moreover, compared to the use of pure biodiesel, the density was significantly decreased when the combustion chamber was pumped by biodiesel-DF blends. However, the density values of the biodiesel-DF blends remained higher than those of DF. Several previous experiments showed that both the blend method and the heating method can be applied simultaneously during operation processes. Engine types and configurations are typical examples of cases where combined heat (e.g., exhaust gas) and electrical power are the heating method (Hoang, 2018). Biodiesel and biodiesel-DF blends can probably be used at lower injected temperatures (Marengo et al., 2009; Le et al., 2017). Table 3, Figure 2, and Figure 3 showed the most relevant parameters in terms of the relationships of density-fraction and density-temperature. In study of Montgomery et al. (2012), a fit criterion was needed when the mean values of the statistical model were used to determine and adjust the regression correlation of the experimental data. For the F-test approach, the statistical model was significant only if the confidence level reached at least 95% (Kanaveli et al., 2017). The results shown in Figures 2 and 3 indicate that the relationships between density and temperature and density and fraction can be considered the mean values of a straight line with high confidence levels of around 99.79% and 98.44%, respectively. The small percentage difference obtained above implies that Equation 7 and Equation 8 can be utilized for the prediction and estimation of the fraction-based density and the temperature-based density of the biodiesel blends. The comparison of the experimental results with those obtained by calculation using Equation 8 showed that the maximum absolute percentage differences in COB, JOB, and WOB were 0.3055%, 0.1871%, and 0.4026%, respectively. There was a small difference between the results of Equation 7 and Equation 8 and the empirically measured values, which was due to a small difference in the densities of the three biodiesel samples.

The calculated results by Equation 7 showed that the absolute error for the estimation of the dependence of biodiesel density on biodiesel fraction was only 0.4956%. The maximal error when used Equation 8 to predict the relationship between biodiesel density and temperature was 0.4026%. Because these errors were exceedingly small (<5%), they may be acceptable for estimating and predicting biodiesel density according to both fraction and temperature.

3.2. The Relationship between Biodiesel KV, Biodiesel Fraction and Temperature

The KV of each biodiesel was estimated to determine an appropriate value related to the requirement of fuel used for diesel engines, this requirement aimed to ensure a good atomization and a fast evaporation of fuel when injected into combustion chamber to the form homogeneous mixture. Figure 4 showed the dependence of the KV of biodiesel blend on the biodiesel fraction.

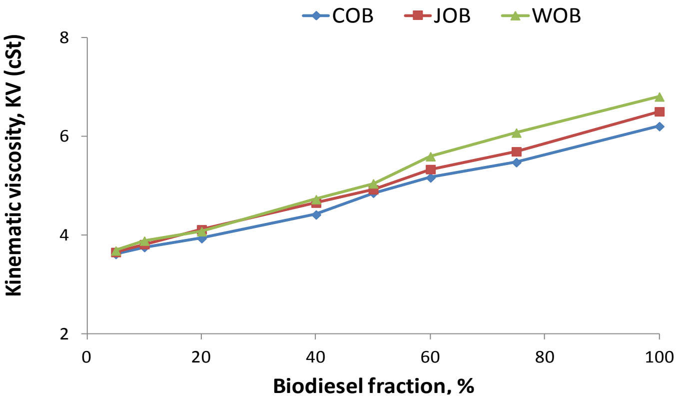

Figure 4 Dependence of KV on biodiesel fraction

The proportional relationship between the KV and the biodiesel fraction is shown in Figure 4. The increase in the biodiesel fraction increased the KV. The experimental results showed that the recommended equation was the second-degree one, Equation 9. The empirical coefficients A, B, and C and the R^2 value were the parameters used to calculate the KV of the biodiesel blends. Table 4 shows the values of KV_1, KV_2, KV_3. Kinematic viscosity (KV_1) and kinematic viscosity (KV_3) were calculated by Equation 4 and Equation 9, respectively. Kinematic viscosity (KV_3) was measured experimentally. Moreover, (Δ_3) was the absolute error of KV, which was determined by empirical Equation 9 and the measured parameters. The results of empirical Equation 9 and calculated Equation 4 yielded the absolute error (Δ_4). These absolute errors are shown in Table 4.

$$KV = Ax^2 + Bx + C \tag{9}$$

Table 4 KV values of biodiesel blends at 30°C

Biodiesel types	Biodiesel fraction (%)	A	B	C	R^2	KV_1, cSt	KV_2, cSt	KV_3, cSt	Δ_3, %	Δ_4, %
COB	5	2.10^{-5}	0.0259	3.4573	0.9952	3.5869	3.61	3.6015	0.6399	0.4056
	10					3.7165	3.75	3.7060	0.8933	0.2845
	20					3.9757	3.93	3.9240	1.1628	1.3167
	40					4.4941	4.41	4.3994	1.9070	2.1516
	50					4.7533	4.85	4.6583	1.9938	2.0388
	60					5.0125	5.16	4.9324	2.8585	1.6230
	75					5.4013	5.47	5.3742	1.2559	0.5050
	100					6.0493	6.20	6.2000	2.4306	2.4306
JOB	5	2.10^{-5}	0.0278	3.5235	0.9988	3.6626	3.65	3.6100	0.3452	1.4563
	10					3.8017	3.81	3.7235	0.2178	2.0999
	20					4.0799	4.11	3.9613	0.7324	2.9941
	40					4.6363	4.65	4.4834	0.2946	3.4107
	50					4.9145	4.92	4.7697	0.1118	3.0359
	60					5.1927	5.33	5.0743	2.5760	2.3335
	75					5.6100	5.69	5.5680	1.4060	0.7537
	100					6.3055	6.50	6.5000	2.9923	2.9923
WOB	5	2.10^{-5}	0.0317	3.4989	0.9943	3.6575	3.68	3.6182	0.6114	1.0868
	10					3.8161	3.88	3.7403	1.6469	2.0252
	20					4.1333	4.07	3.9972	1.5553	3.4048
	40					4.7677	4.72	4.5650	1.0106	4.4394
	50					5.0849	5.04	4.8785	0.8909	4.2303
	60					5.4021	5.59	5.2135	3.3614	3.6168
	75					5.8779	6.07	5.7597	3.1647	2.0525
	100					6.6709	6.80	6.8000	1.8985	1.8985

Table 4 and Figure 4 show three different results of the experiments and the calculations. When the biodiesel faction was fixed in the range from 5%–100%, the KVs of the biodiesel blends were from 3.61 cSt–6.80cSt in the experimental case, from 3.5869 cSt–6.6709 cSt in the results of Equation 9, and from 3.6015cSt to 6.80cSt in the results of Equation 4. The comparison of Equation 9 and Equation 4 showed that the value of 4.4394% was the maximum absolute error. The maximum value was 3.36% in the results of the empirical Equation 10. The correlation shown by a second-degree equation with $R^2 \approx 1$ was acceptable with high reliability because this maximum absolute error was lower than 5%. Because of its suitable accuracy, Equation 9 can be used to estimate the KV of biodiesel blends if pure biodiesel KV and diesel fuel are clearly distinguished. Figure 5 shows that the temperature depends on the KV of biodiesel in a temperature range of 30°C–80°C

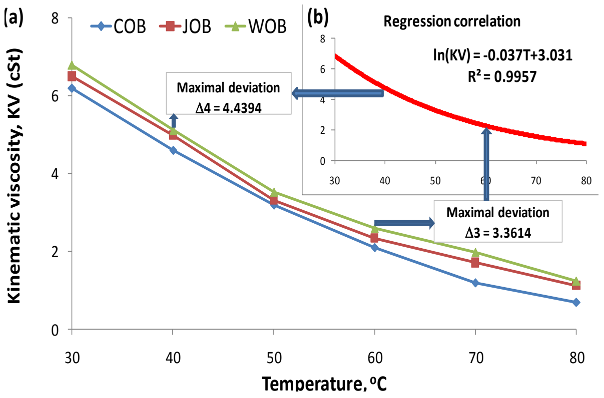

Figure 5 The dependence of biodiesel KV on temperature

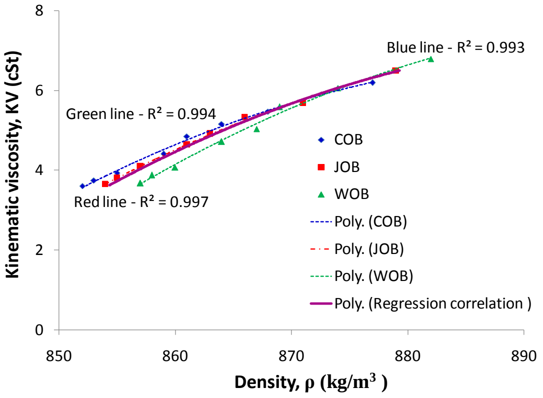

Figure 6 The function of KV and density with fraction variations

Figure 5 shows the empirical correlation between the KV and the heating temperature. Similar to Equation 10, it can be described in a logarithmic function with an R^2 value of 0.9957. The obtained Equation 10 is similar to those used in previous studies (Maria et al., 2011; Kanaveli et al., 2017).

$$\ln(KV) = -0.037T + 3.031 \tag{10}$$

Table 4 and Figure 5 show that the limited confidence level of 95% was similar to that described in section 3.1. Therefore, a second-degree equation was used for the estimation of the fraction-based KV function; the logarithmic function was more suitable to calculate the temperature-based KV correlation. Furthermore, a mathematical expression or equation was useful in determining the link between the two key parameters of biodiesel: density and KV. Based on this approach, the determination of biodiesel KV is simple and fast if biodiesel density is known. The biodiesel KV–density relationship depended on the biodiesel fraction shown in Figure 6. Figure 7 shows that the biodiesel KV–density relationship depended on temperature. Based on the experimental results and the newly calculated correlation, the maximal errors were around 4.44% in Equation 9 and 3.36% in Equation 10. In addition, the high value of R^2 (around 99%) showed the high reliability of Equation 9 and Equation 10 for the estimation of the dependence of biodiesel kinematic viscosity on fraction and temperature, respectively.

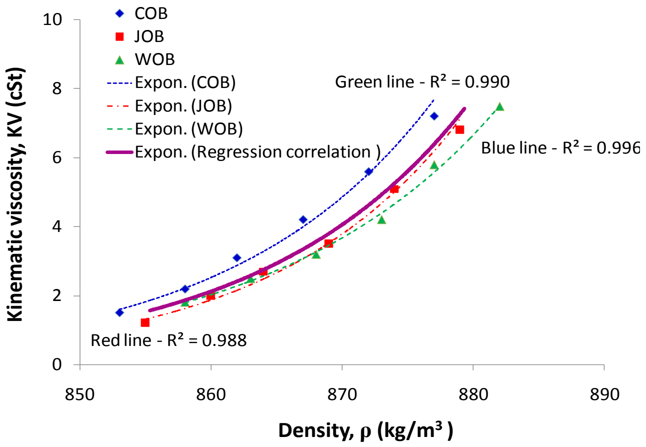

Figure 7 KV and density of three studied biodiesel types with temperature variation

The KVs of the diesel–biodiesel blends were characterized as a function of their density values between fraction and temperature variation. The results of a regression correlation are shown in Figure 6 and Figure 7. These results were used in the correlation equations Equation 11 and Equation 12. The results shown in Figure 6 and Figure 7 were favorable with confidence levels of $R^2 = 0.9961$ and $R^2 = 0.9978$, respectively,

$$KV = -0.017\rho^2 + 3.0034.\rho - 1346.2 \tag{11}$$

$$KV = 3.68.10^{-28}\exp(0.074.\rho) \tag{12}$$

The results of Equation 11 and Equation 12 showed good estimations of the KV of the diesel–biodiesel blend. The density of the diesel–biodiesel blend was measured in both cases—biodiesel fraction variation and temperature variation. The correlations are similar to those in Tesfa et al. (2010).

Equation 11 and Equation 12 show differences in the confidence level R^2. The regression correlation based on the exponent function may be more appropriate to the empirical correlation between biodiesel kinematic viscosity and biodiesel density: $R^2 = 0.9978$. The maximal errors in using Equation 12 to calculate the KV values of the density of biodiesel were compared to the empirical results were lower than 5%. Thus, Equation 12 may be used to obtain an accurate estimation and prediction of the relationship between the kinematic viscosity and density of as-used biodiesel fuel.

4. CONCLUSION

In this work, a dual model based on temperatures and volume fractions to estimate the density and kinematic viscosity of pure biodiesel and biodiesel–diesel fuel blends was developed. Three types of biodiesel, COB, JOB, and WOB, were used in both experiments. The main results are summarized as follows:

The density and kinematic viscosity of pure biodiesel were inversely proportional to temperature, whereas the density and kinematic viscosity of biodiesel–diesel fuel blend increased as the volume fractions of the biodiesel increased. Empirical equations with high confidence levels were established.

The results showed $R^2 = 0.9979$ in predicting the dependence of the density of pure biodiesel on temperature and $R^2 = 0.9844$ in predicting the dependence of the density of biodiesel–diesel fuel blends on volume fractions. Similarly, the results showed $R^2 = 0.9942$ and $R^2 = 0.9957$ in predicting kinematic viscosity. In the experimental results and the model, the relationship between density and kinematic viscosity was $R^2 = 0.9961$.

5. REFERENCES

Abubakar, H., Abdulkareem, A., Jimoh, A., Agbajelola, O., Okafor, J., Afolabi, E., 2016. Optimization of Biodiesel Production from Waste Cooking Oil. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, Volume 38(16), pp. 2355–2361

Albuquerque, M.C.G., Machado, Y.L., Torres, A.E.B., Azevedo, D.C.S., Cavalcante, C.L.-Jr., Firmiano, L.R., Parente, E.J.S.-Jr., 2009. Properties of Biodiesel Oils Formulated using Different Biomass Sources and Their Blends. *Renewable Energy,* Volume 34(3), pp. 857–859

Aliyah, A.N., Edelweiss, E.D., Sahlan, M., Wijanarko, A., Hermansyah, H., 2016. Solid State Fermentation using Agroindustrial Wastes to Produce Aspergillus Niger Lipase as a Biocatalyst Immobilized by an Adsorption-crosslinking Method for Biodiesel Synthesis. *International Journal of Technology,* Volume 7(8), pp. 1393–1404

Anh Tuan, L., Minh Tuan, P., 2009. Impacts of Gasohol E5 and E10 on Performance and Exhaust Emissions of In-used Motorcycle and Car: A Case Study in Vietnam. *Journal of Science and Technology (Technical Universities)*, Volume 73, pp. 98–104

Bhale, P.V., Deshpande, N.V., Thombre, S.B., 2009. Improving the Low Temperature Properties of Biodiesel Fuel. *Renewable Energy*, Volume 34(3), pp. 794–800

Camas-Anzueto, J.L., Gómez-Pérez, J., Meza-Gordillo, R., Anzueto-Sánchez, G., Pérez-Patricio, M., López-Estrada, F.R., Abud-Archila, M., Ríos-Rojas, C., 2017. Measurement of the Viscosity of Biodiesel by using an Optical Viscometer. *Flow Measurement and Instrumentation,* Volume 54, pp. 82–87

CEN., 2008. *Automotive Fuels - Fatty Acid Methyl Esters (FAME) for Diesel Engines - Requirements and Test Methods*. EN 14214:2008, European Committee for Standardization, Brussel, Belgium

Chavarria-Hernandez, J.C., Pacheco-Catalán, D.E., 2014. Predicting the Kinematic Viscosity of Fames and Biodiesel: Empirical Models. *Fuel,* Volume 124, pp. 212–220

Esteban, B., Riba, J.-R., Baquero, G., Rius, A., Puig, R., 2012. Temperature Dependence of Density and Viscosity of Vegetable Oils. *Biomass and Bioenergy,* Volume 42, pp. 164–171

Fasina, O., Colley, Z., 2008. Viscosity and Specific Heat of Vegetable Oils as a Function of Temperature: 35°C to 180°C. *International Journal of Food Properties,* Volume 11(4), pp. 738–746

Gülüm, M., Bilgin, A., 2017. Measurements and Empirical Correlations in Predicting Biodiesel-Diesel Blends' Viscosity and Density. *Fuel*, Volume 199, pp. 567–577

Hoang, A.T., 2017. The Performance of Diesel Engine Fueled Diesel Oil in Comparison with Heated Pure Vegetable Oils Available in Vietnam. *Journal of Sustainable Development,* Volume 10(2), pp. 93–103

Hoang, T.A., Le, V.V., 2017. The Performance of a Diesel Engine Fueled with Diesel Oil, Biodiesel and Preheated Coconut Oil. *International Journal of Renewable Energy Development*, Volume 6(1), pp. 1–7

Hoang, A.T., Nguyen, V.T., 2017. Emission Characteristics of a Diesel Engine Fueled with Preheated Vegetable Oil and Biodiesel. *Philippine Journal of Science*, Volume 146(4), pp. 475–482

Hoang, A.T., 2018. Waste Heat Recovery from Diesel Engines Based on Organic Rankine Cycle. *Applied Energy*, Volume 231, pp. 138–166

Hoang, A.T., Le, A.T., 2018. A Review on Deposit Formation in the Injector of Diesel Engines Running on Biodiesel. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*. Available online at: https://doi.org/10.1080/15567036.2018.1520342, Accessed on 22 September 2018

Hoang, A.T, Noor, M.M, Pham, X.D., 2018. Comparative Analysis on Performance and Emission Characteristic of Diesel Engine Fueled with Heated Coconut Oil and Diesel Fuel. *International Journal of Automotive & Mechanical Engineering,* Volume 15(1), pp. 5110–5125

Hoang, A.T., Pham, V.V., 2018. A Study of Emission Characteristic, Deposits, and Lubrication Oil Degradation of a Diesel Engine Running on Preheated Vegetable Oil and Diesel Oil. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*. Available online at: https://doi.org/10.1080/15567036.2018.1520344, Accessed on 26 September 2018

Kanaveli, I.-P., Atzemi, M., Lois, E., 2017. Predicting the Viscosity of Diesel/Biodiesel Blends. *Fuel*, Volume 199, pp. 248–263

Le Anh Tuan, Pham Huu Tuyen, Van Dinh Son Tho. 2017. Alternative fuels for internal combustion engine. Bach khoa Publishing House

Leong, S.K., Lam, S.S., Ani, F.N., Ng, J.-H., Chong, C.T., 2016. Production of Pyrolyzed Oil from Crude Glycerol using a Microwave Heating Technique. *International Journal of Technology*, Volume 7(2), pp. 323–331

Liu, R., Li, C., Zhang, H., Xiao, Z., Zhang, A., Chen J., 2017. Pilot-scale Study of Esterification of Waste Oil for Biodiesel Production. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects,* Volume 39(1), pp. 29–35

Marengo, E., Longo, V., Bobba, M., Robotti, E., Zerbinati, O., Di Martino, S., 2009. Butene Concentration Prediction in Ethylene/Propylene/1-Butene Terpolymers by FT-IR Spectroscopy through Multivariate Statistical Analysis and Artificial Neural Networks. *Talanta*, Volume 77(3), pp. 1111–1119

Montgomery, D.C., Peck, E.A., Vining, G.G. 2012. *Introduction to Linear Regression Analysis.* John Wiley & Sons, Inc., Hoboken, New Jersey, USA

Rajagopal, K., Bindu, C., Prasad, R., Ahmad, A., 2016. The Effect of Fatty Acid Profiles of Biodiesel on Key Fuel Properties of Some Biodiesels and Blends. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, Volume 38(11), pp. 1582–1590

Saxena, P., Jawale, S., Joshipura, M.H., 2013. A Review on Prediction of Properties of Biodiesel and Blends of Biodiesel. *Procedia Engineering*, Volume 51, pp. 395–402

Suryantoro, M.T., Sugiarto, B., Chistian, D., Samudra, B., Gusfa, Z., 2016. Deposit Characterization of a Diesel Engine Combustion Chamber by Droplets at Hot Chamber Temperature: Effect of Temperature on Evaporation Time and Deposit Structure. *International Journal of Technology,* Volume 7(8), pp. 1373–1381

Tat, M.E., Van Gerpen, J.H., 2000. The Specific Gravity of Biodiesel and Its Blends with Diesel Fuel. *Journal of the American Oil Chemists' Society*, Volume 77(2), pp. 115–119

Tesfa, B., Mishra, R., Gu, F., Powles, N., 2010. Prediction Models for Density and Viscosity of Biodiesel and Their Effects on Fuel Supply System in CI Engines. *Renewable Energy,* Volume 35(12), pp. 2752–2760

Tran, V.D., Le, A.T., Dong, V.H., Hoang, A.T., 2017. Methods of Operating the Marine Engines by Ultra-Low Sulfur Fuel to Aiming to Satisfy MARPOLAnnex VI. *Advances in Natural and Applied Sciences*, Volume 11(12), pp. 34–40

Yang, C., He, K., Xue, Y., Li, Y., Lin, H., Sheng H., 2018. Factors affecting the Cold Flow Properties of Biodiesel: Fatty Acid Esters. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects,* Volume 40(5), pp. 516–522

Yang, C., Zhang, B., Cui, C., Wu, J., Ding, Y., Wu Y., 2016. Standards and Protocols for Characterization of Algae-Based Biofuels. *Trends in Renewable Energy,* Volume 2(2), pp. 56–60